Natural Fourier Transform

Emergent Harmonic Synthesis in Plucked String Vibrations

Dr. Anando Gopal Chatterjee - 29/01/2026

Outline

- The Beginning of FFT
- My Journey through FFT
- Scaling studies on Shaheen, KAUST
- Network Topology
- Best Hardware
- FFT in Nature

The beginning

Heat Equation

Jean-Baptiste Joseph Fourier developed the Fourier transform concept in the early 1800s while studying heat conduction

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2},$$
$$u(x,0) = \phi(x) .$$

Using the integrating factor $e^{\alpha k t}$

Evolved into the Fourier Transform Kernel

Where:

u: is the temperature

α: is the thermal diffusivity,

k: is the wave number, and

t: is the time evolved

Fast Fourier Transform: The Beginning

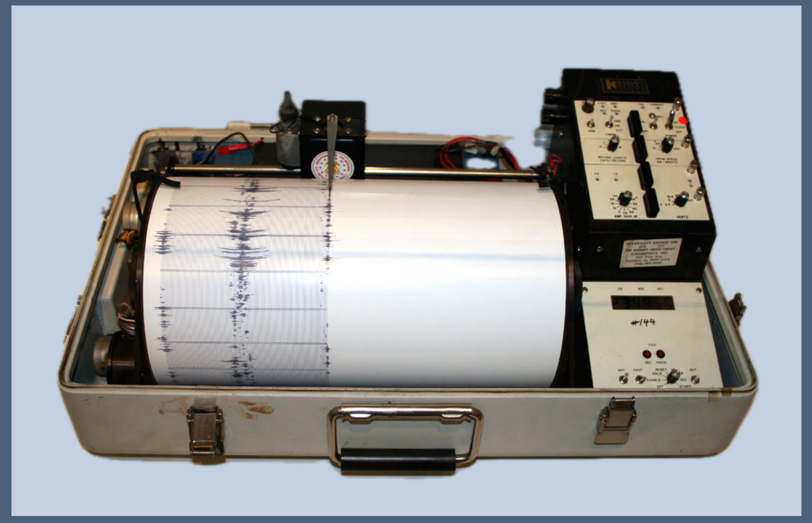
1945

Hiroshima/Nagasaki

Post-1945

Treaty signed; compliance issues

1963



FFT conception at Kennedy SAC

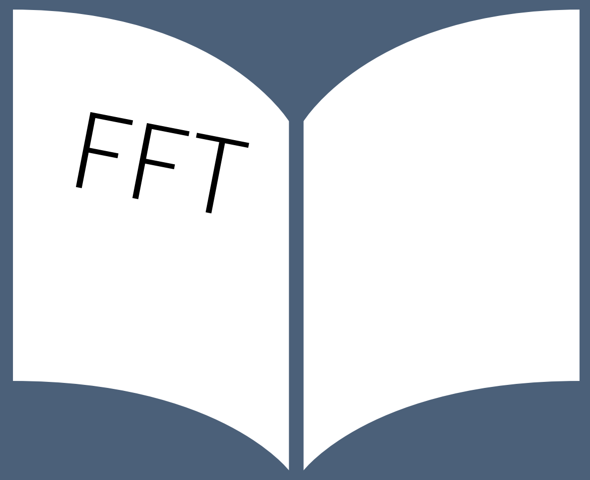
1963-64

Tukey meets Cooley

1964

IBM Watson demo

1965



Publication

Outcome

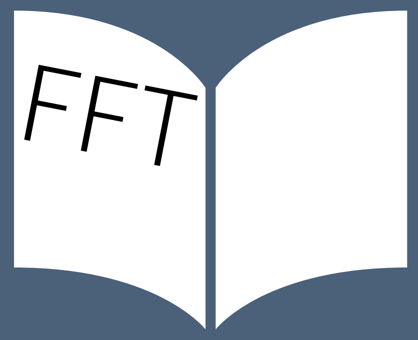
Rapid analysis for verification

The Most Important Algorithm Of All Time, Veritasium (Youtube), 2022, https://www.youtube.com/watch?v=nmgFG7PUHfo

Finite Fourier Transform theory and its application to the computation of convolutions, correlations, and spectra. Unclassified AD800371. 1966

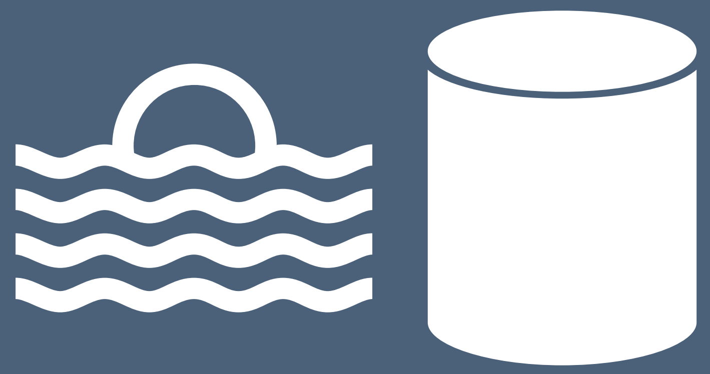
FFT in Signal Processing

1965



FFT Published
4000× speedup

1966–67

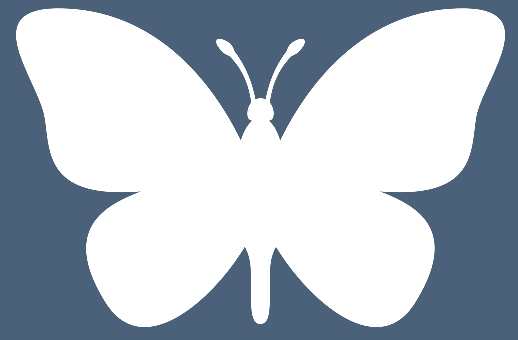


Seismic study and
Oil prospecting adoption

Late 1960s

Speech engineers
lead popularisation

1970s



Dedicated FFT processors

1970s+

Radar, telecom,
SETI applications

My Journey through FFT

Tarang

- a Pseudo-spectral fluid solver by Prof. M K Verma Group

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{F}$$

$$\nabla \cdot \mathbf{u} = 0$$

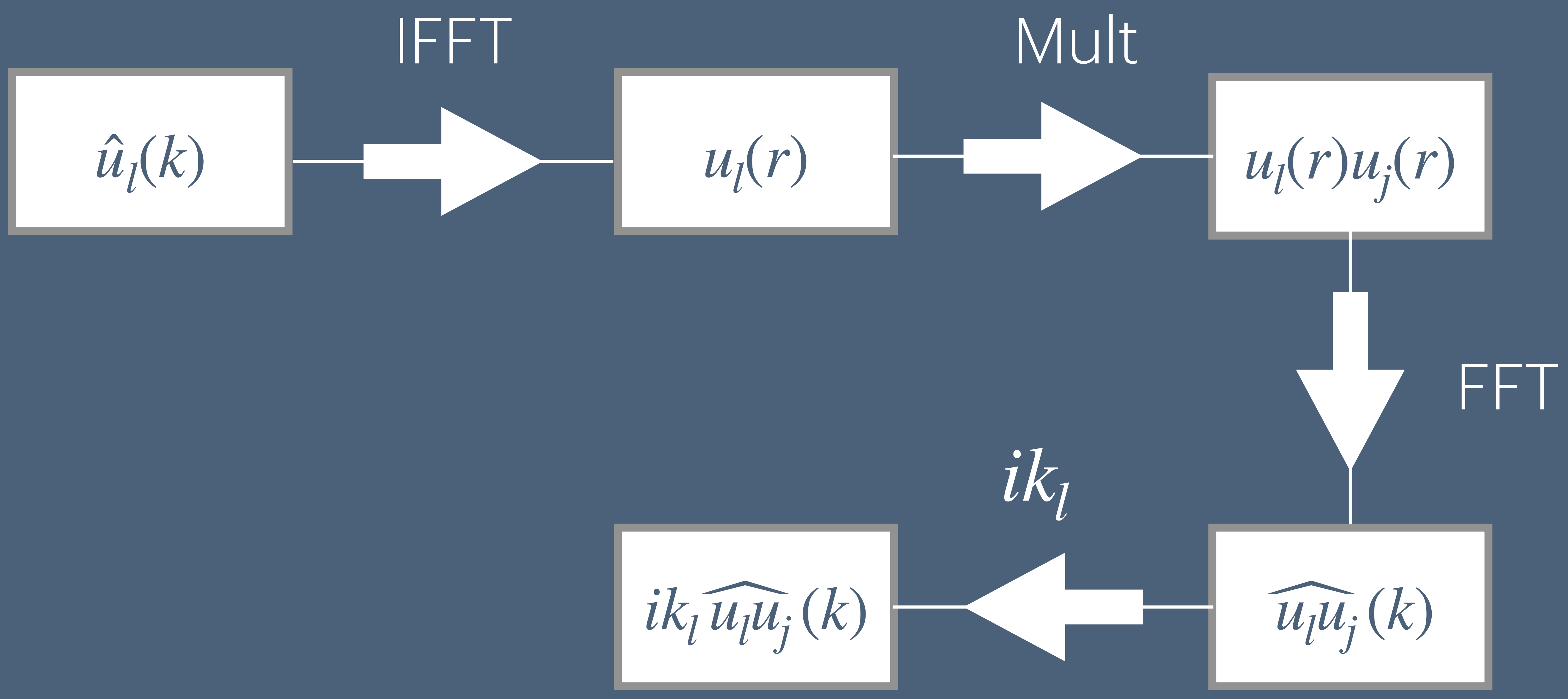
M. K. Verma, A. Chatterjee, and S. Reddy,
Object-oriented Pseudo-spectral code TARANG for turbulence simulation,
In proceedings “ATIP/A*CRC Workshop on Accelerator Technologies for High-Performance Computing: Does Asia Lead the Way?”,
Singapore, p. 4 (2012).

Scaling of Fast Fourier Transform

- Pseudo-spectral method is widely used Fluid Dynamics due to it's High spatial accuracy.

$$\partial_t u_j(\mathbf{k}) = -ik_l \widehat{u_l u_j}(\mathbf{k}) - ik_j p(\mathbf{k}) - \nu k^2 u_j(\mathbf{k})$$

$$k_j u_j(\mathbf{k}) = 0$$



In Spectral Approach:

Computations Required: N^3

In FFT Approach (pseudo-spectral):

Computations Required: $Nlog_2N$

Communications Required: n^2

MPI Function: MPI_Alltoall

N: Grid Size

n: no. of nodes

Discrete Fourier Transform

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}, \quad k = 0, \ldots, N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}, \quad n = 0, \ldots, N-1$$

Real to Complex $N/2 + 1$ MPI_Alltoallv

RBC and Stratified Flows

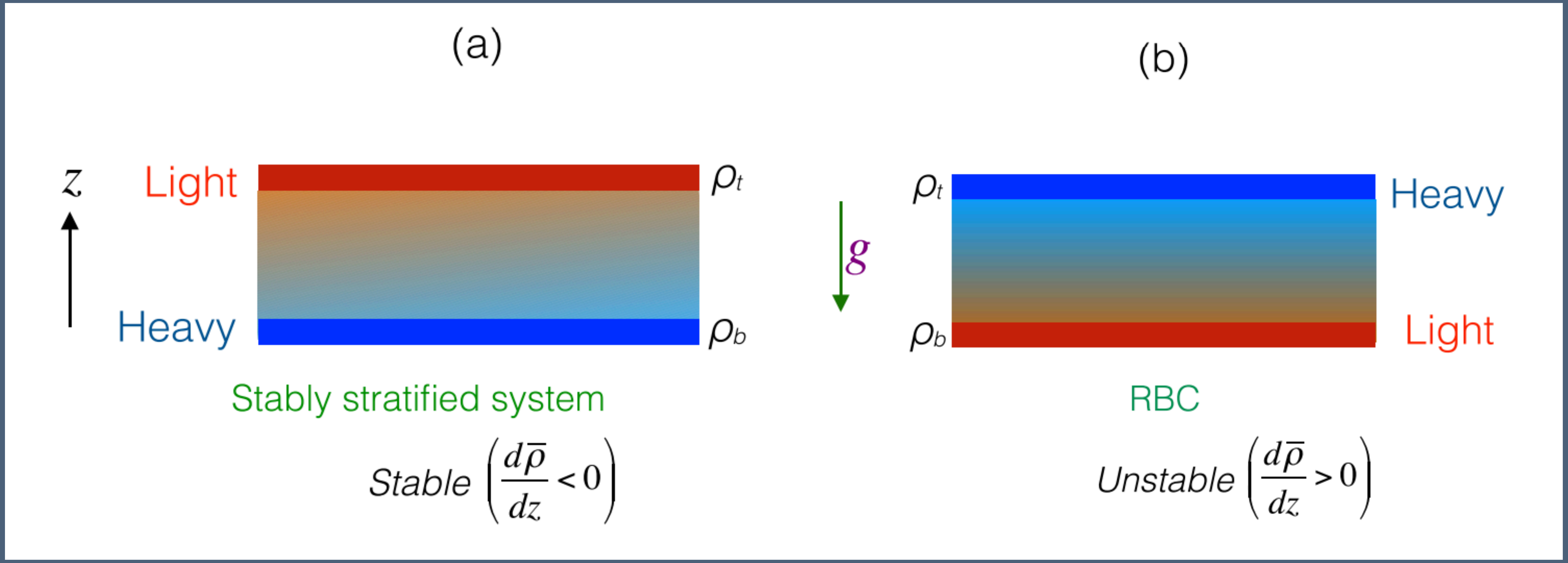
Rayleigh-Bénard convection

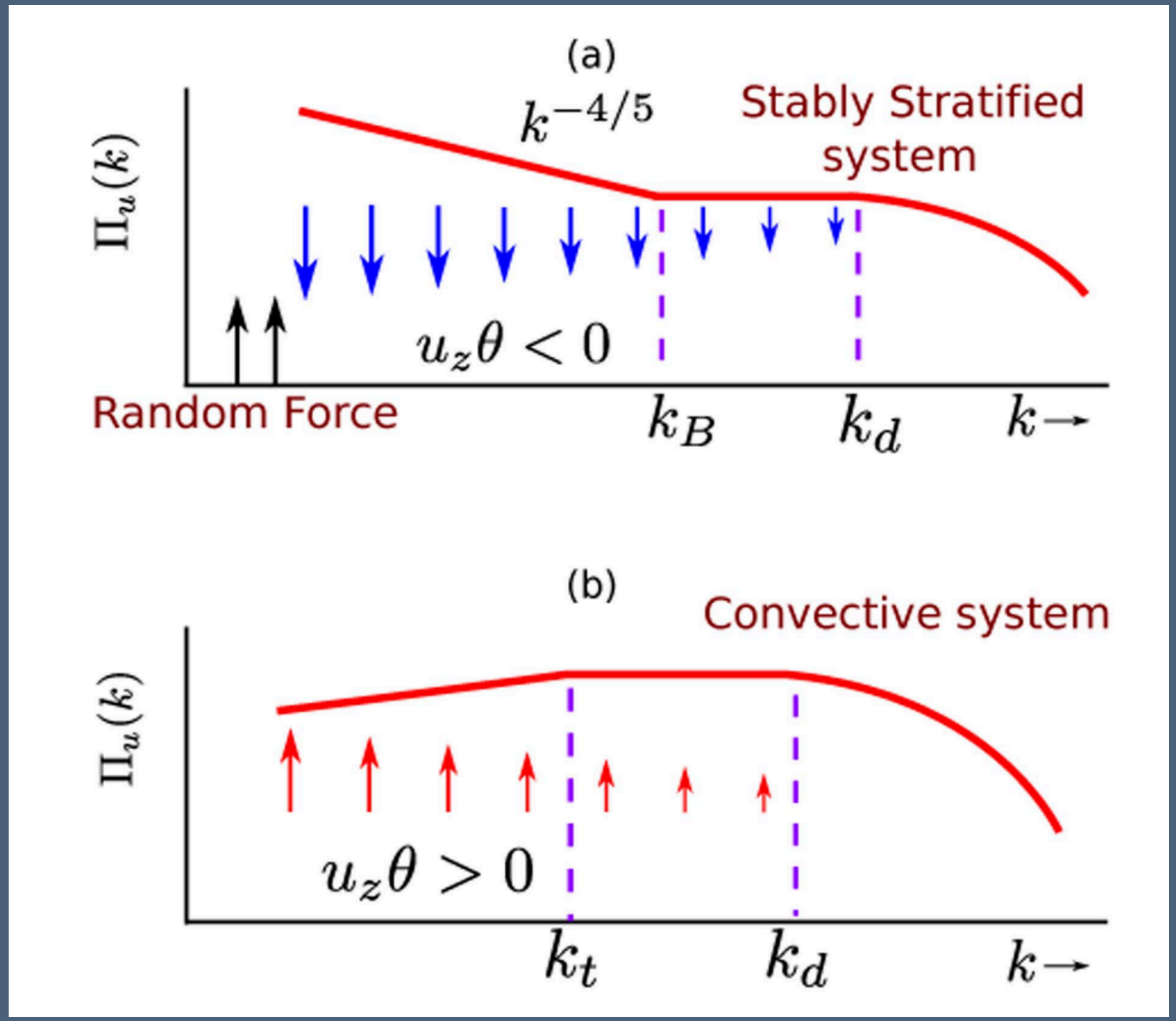
Stratified Flows

Advantages of Pseudo-Spectral Studies

Kolmogorov's Equation: $E(k) \sim k^{-5/3}$

Bolgiano-Obukhov $E(k) \sim k^{-11/5}$





There are two RBC and two Stably Stratified layers in the atmosphere

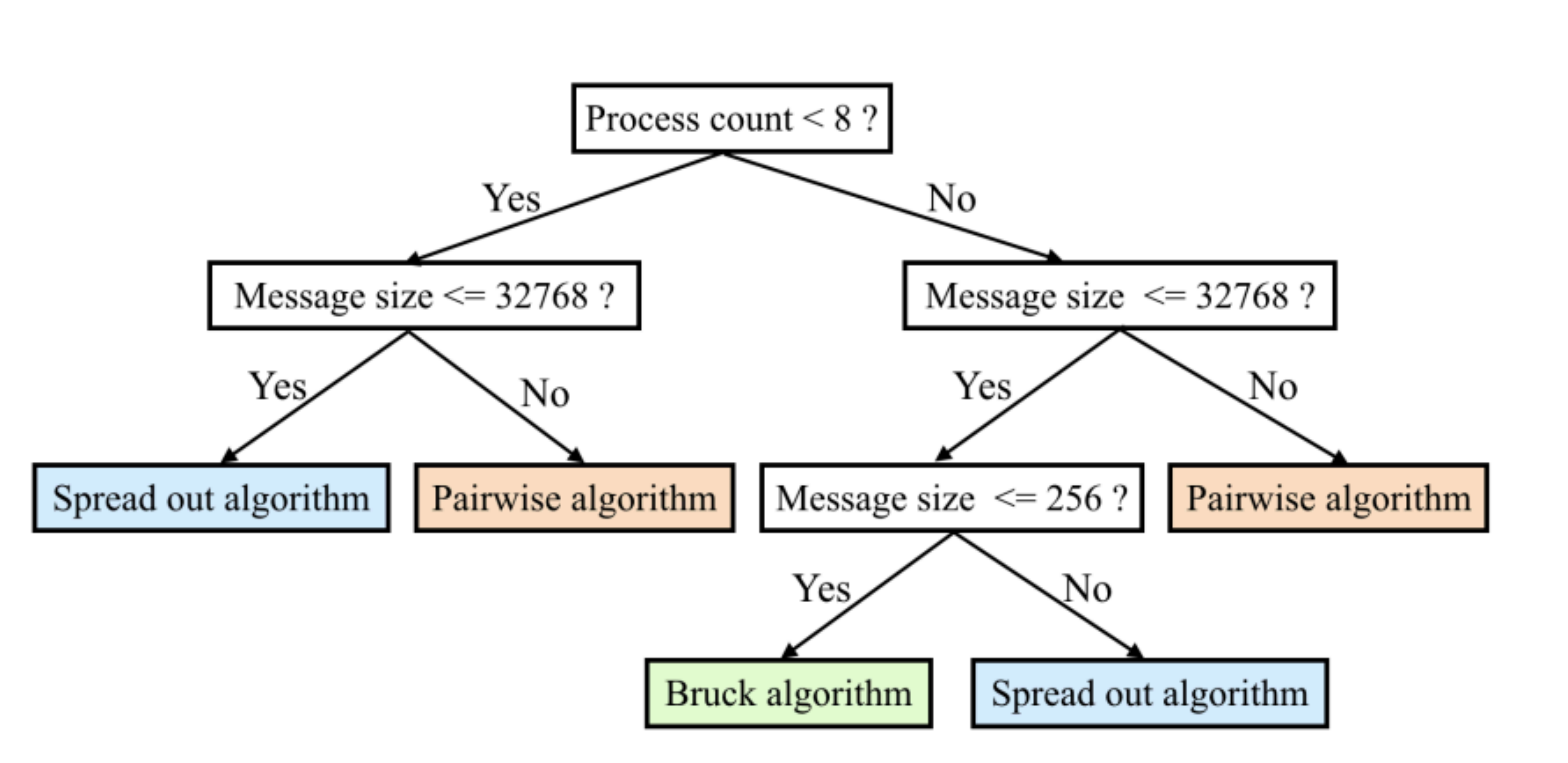
Resolved turbulent fluids have zero energy at highest modes

M. K. Verma, A. Kumar, and A. Pandey, *Phenomenology of buoyancy-driven turbulence: recent results*, New J. Phys., **19**, 025012 (2017).

Performance of MPI functions

MPI Function	Relative Performance	Best For	Overhead
MPI_Send - MPI_Recv	Fast	Point-to-point data exchange	Low for small data
MPI_Bcast	Fast	Synchronizing single data point	Moderate
MPI_Scatter	Moderate	Distributing unique data to each process	Moderate
MPI_Gather	Moderate to High	Collecting data back to one process	High
MPI_Alltoall	Slow	Exchanging data between all processes	Very High
MPI_Allreduce	Fast	Aggregating results across processes	Low to Moderate
MPI_Reduce	Moderate	Summarizing results to one process	Moderate

MPI Alltoall

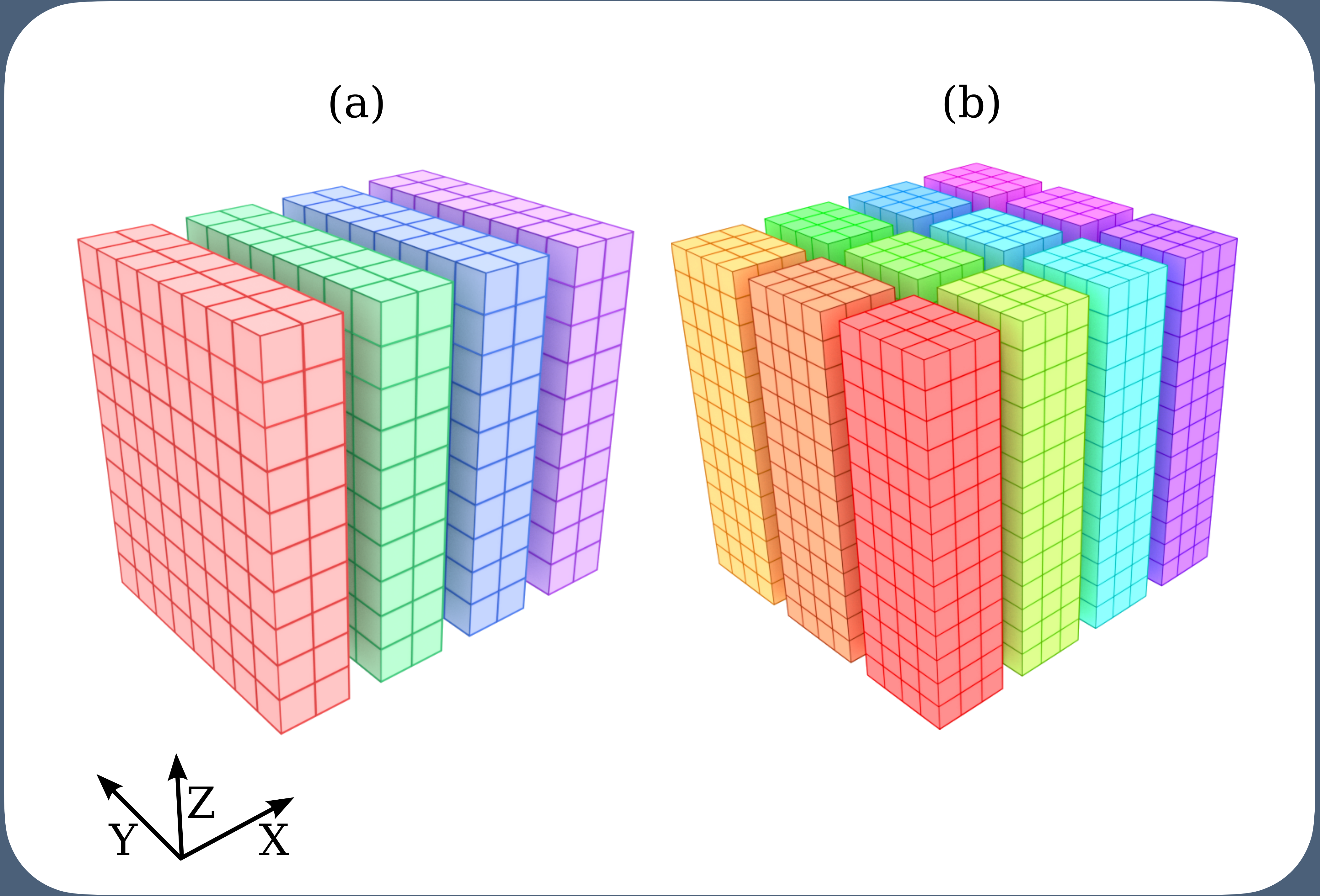


- Bruck algorithm
  $log_2 n$ Communications
- Spread out Algorithm
  n Communications
- Pairwise Algorithm
  n^2 Communications

N. Netterville, K. Fan, S. Kumar and T. Gilray, "A Visual Guide to MPI All-to-all," 2022 IEEE 29th International Conference on High Performance Computing, Data and Analytics Workshop (HiPCW), Bengaluru, India, 2022, pp. 20-27
doi: 10.1109/HiPCW57629.2022.00008.

Scaling studies on Shaheen, KAUST

Data Parallelism



$$T = \frac{N}{B} = c_1 N \left(\frac{1}{p^{\gamma_1}}\right) + c_2 N \left(\frac{1}{n^{\gamma_2}}\right)$$

We have developed an FFT library named FFTK (FFT Kanpur)

We use FFTW for 1D Transforms

Since energy in high modes are zero,
we ignore the last mode in complex plane $(N/2+1)^{\text{th}}$
and use MPI_Alltoall for 2D/3D decomposition

Young Researcher Award by InSc (2023)

A. G. Chatterjee, M. K. Verma, A. Kumar, R. Samtaney, B. Hadri, and R. Khurram,
Scaling of a Fast Fourier Transform and a pseudo-spectral fluid solver up to 196608 cores, J. Parallel Distrib. Comput., 113, 77-91 (2017)

Various Optimisation Attempts

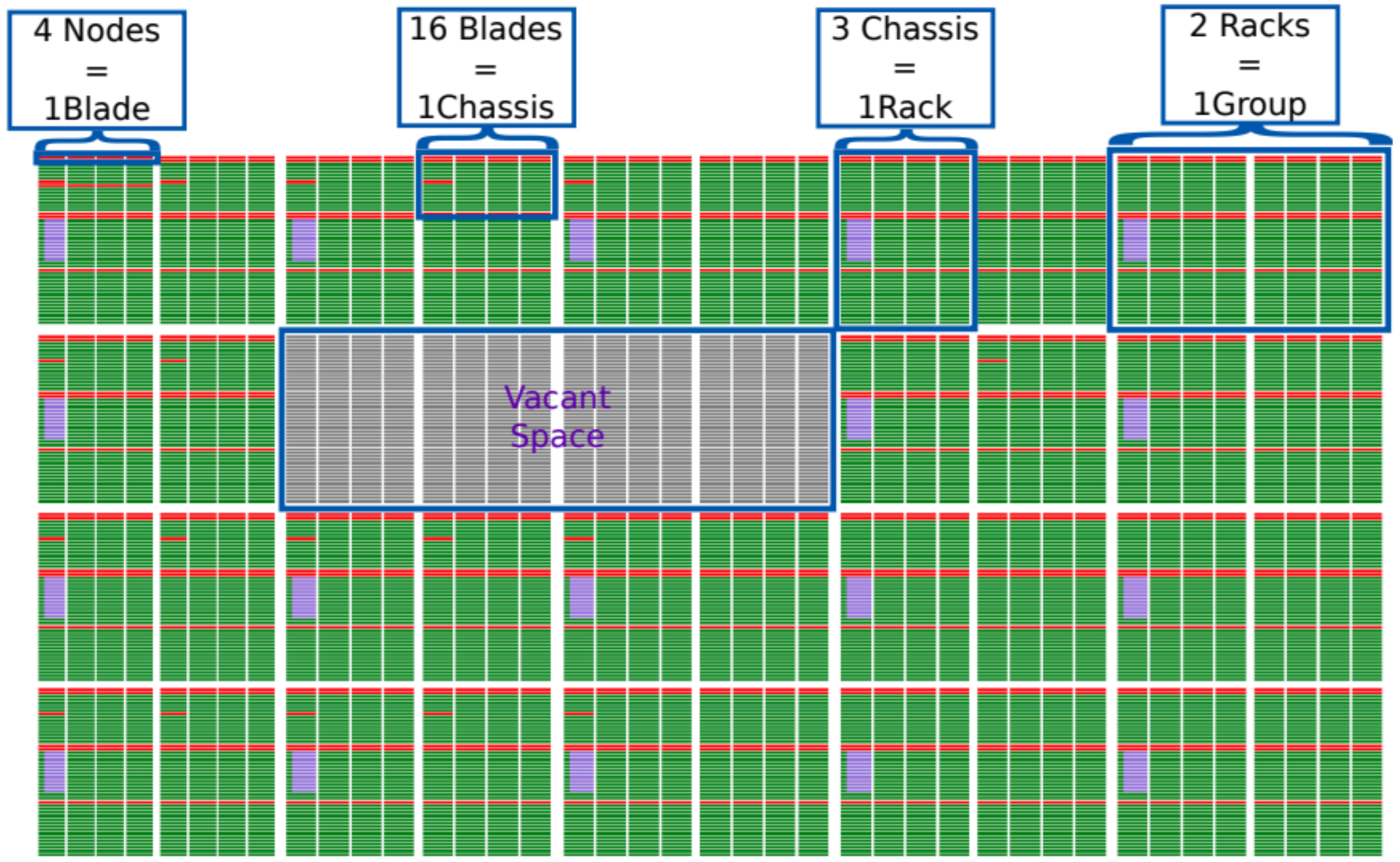


FIGURE 2 Schematic representation of Shaheen's physical structure displaying it's `blades`, `chassis`, `racks` and `groups`. Here, the red nodes are service nodes, green are compute nodes, and violet nodes are used for an experiment described in Sec. 4.5.

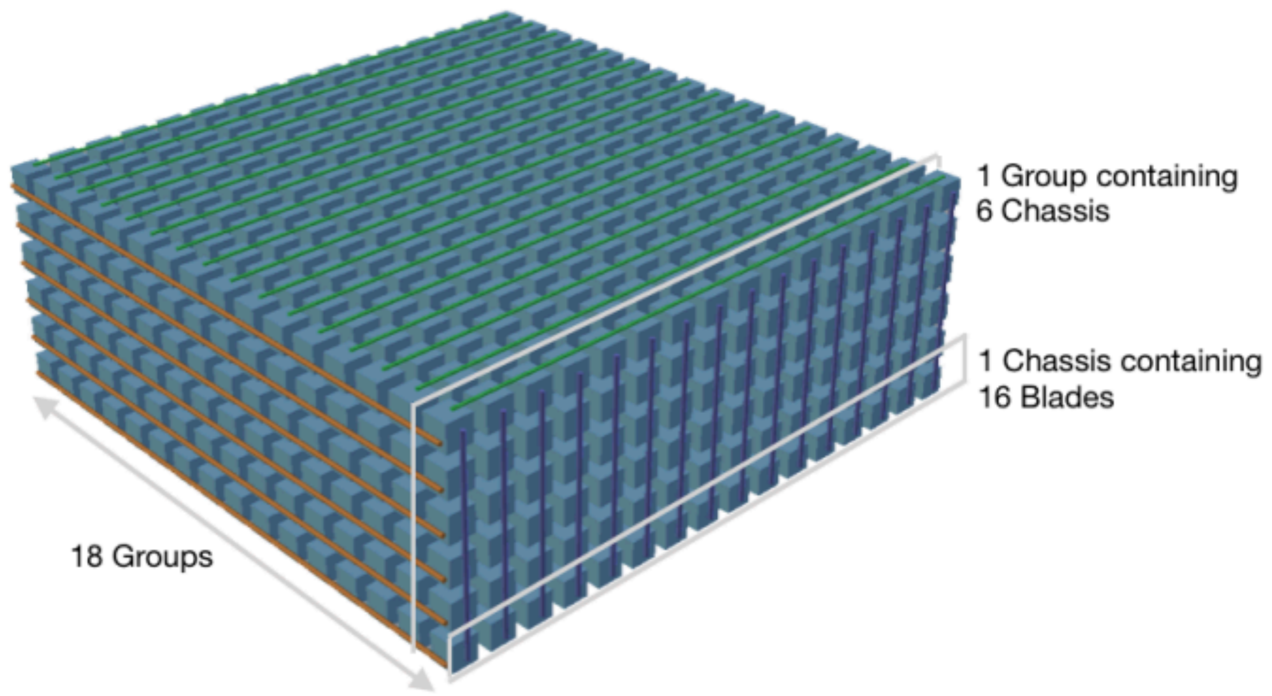
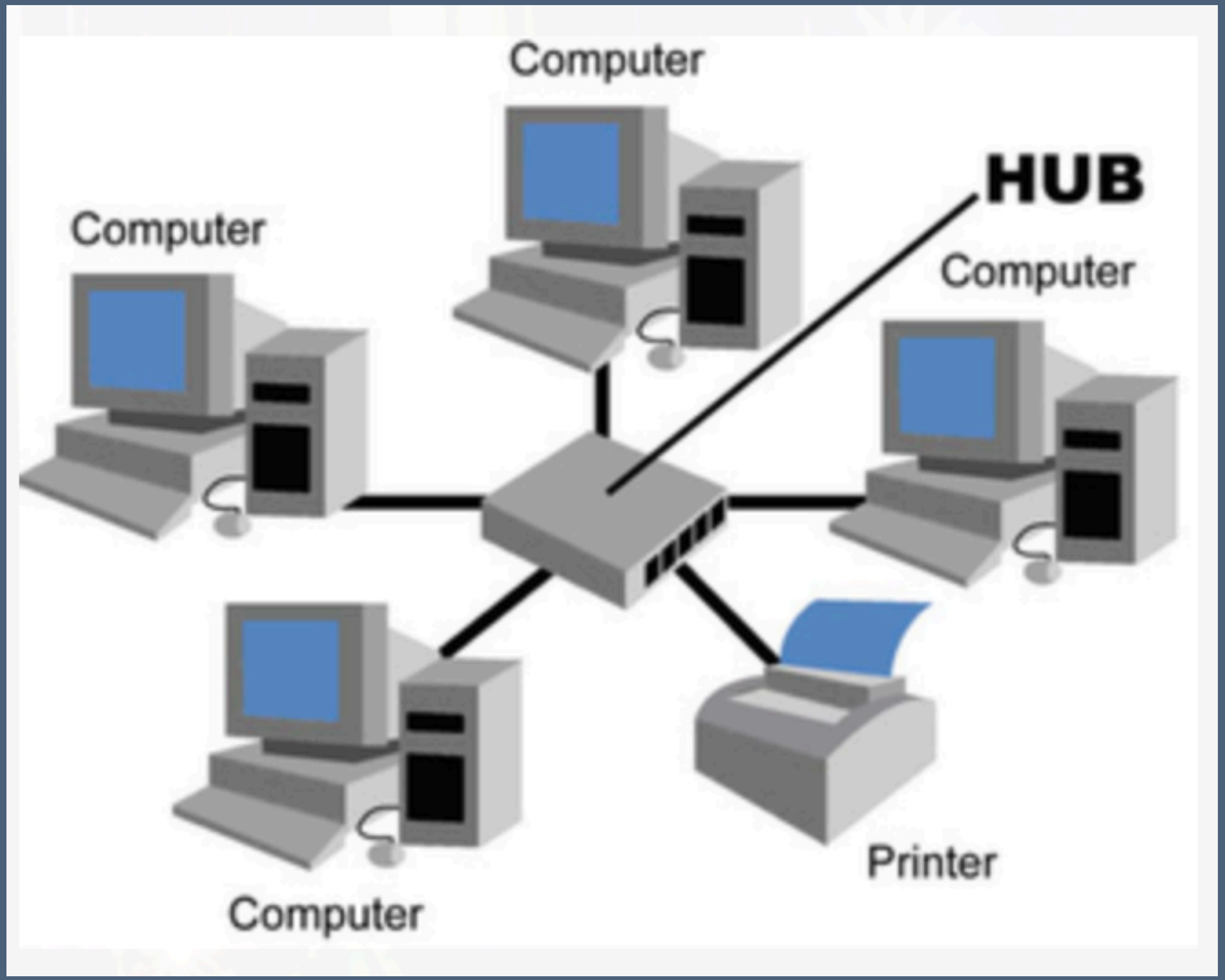


FIGURE 3 Schematic representation of Shaheen's topology. Each cube represents a blade.

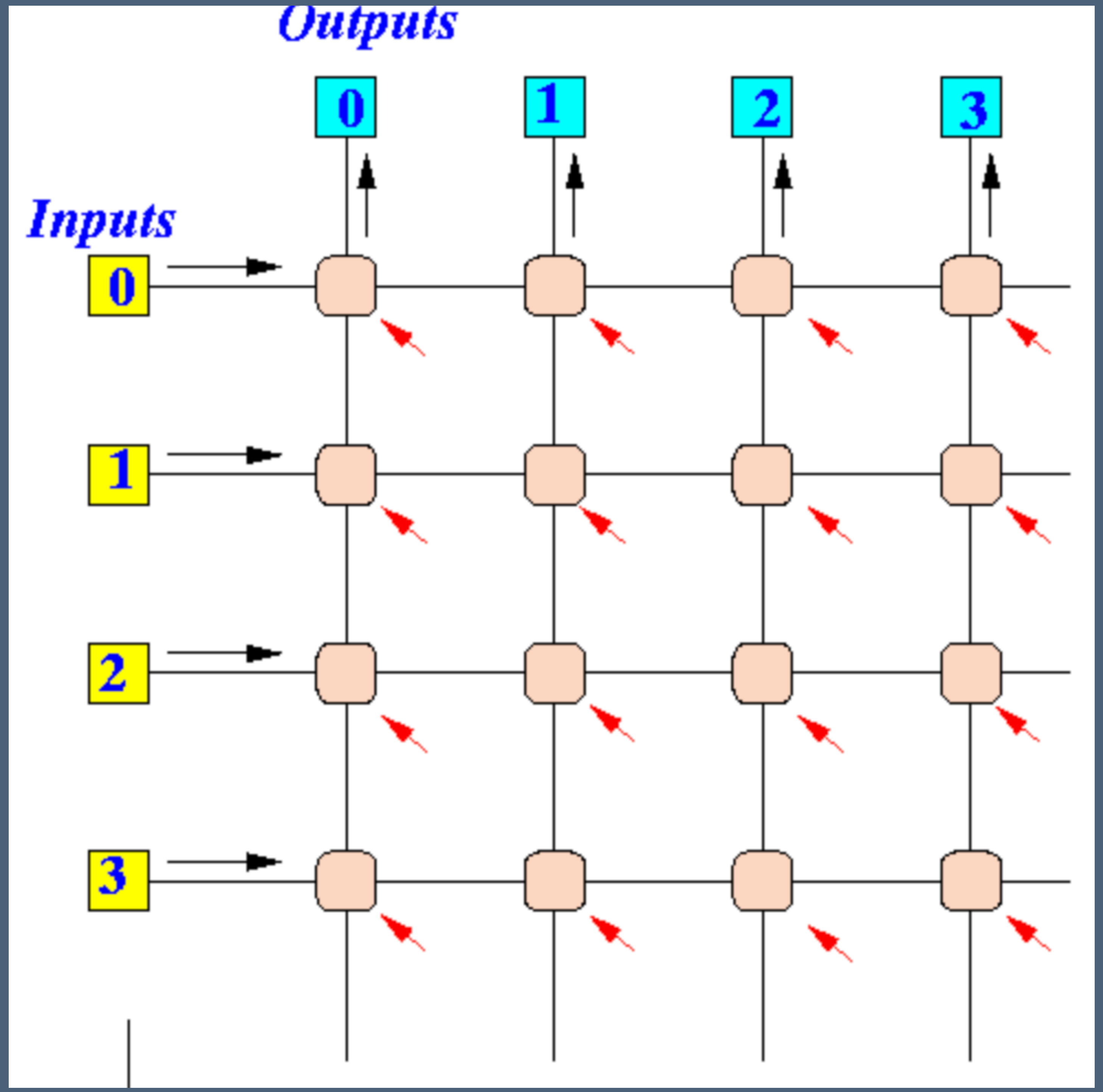
Aseeri, S. A., Gopal Chatterjee, A., Verma, M. K., & Keyes, D. E. (2021). *A scheduling policy to save 10% of communication time in parallel fast Fourier transform.* Concurrency and Computation: Practice and Experience. doi:10.1002/cpe.6508

Network Topology

Star Topology



Regular routers/switches

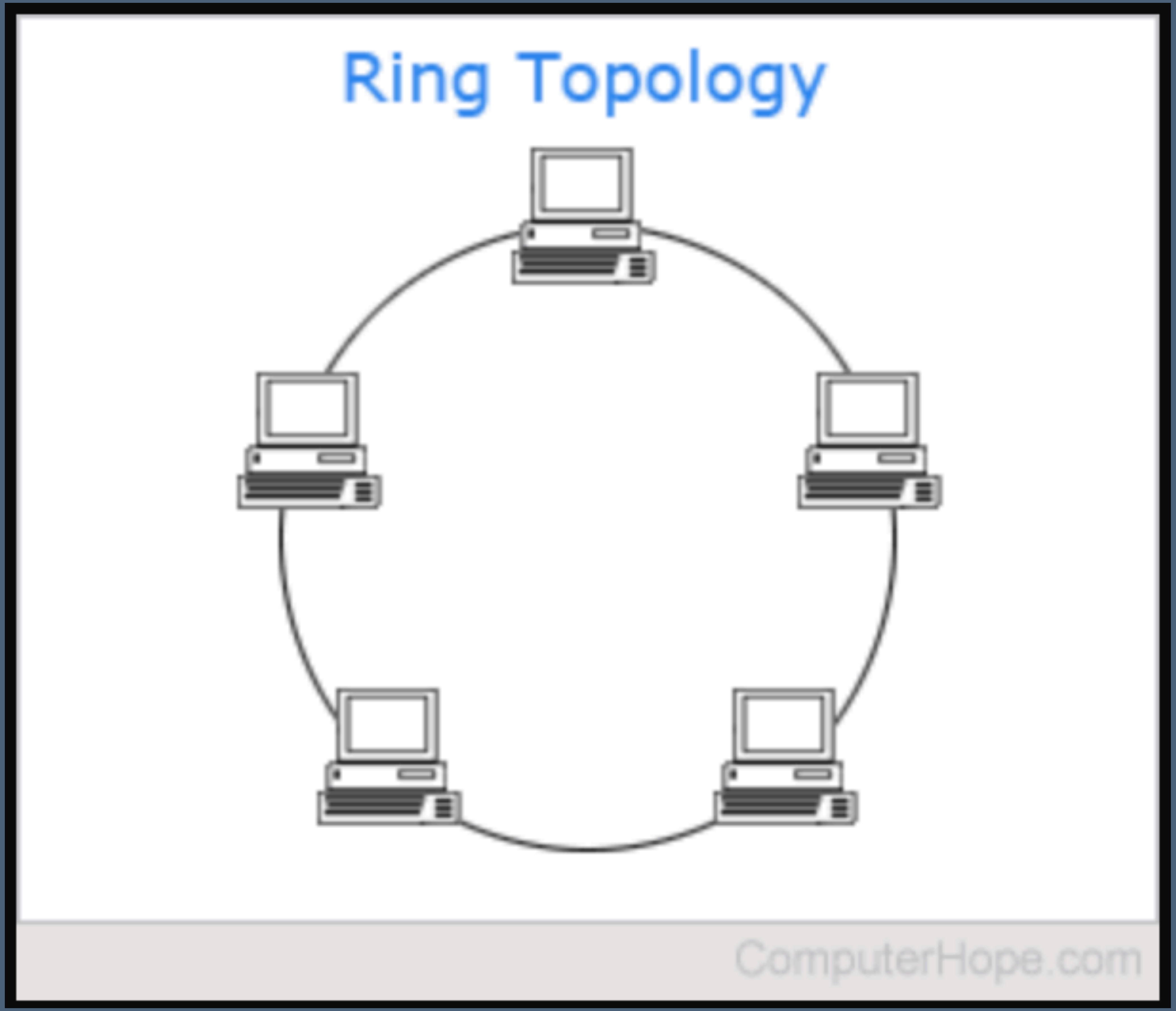


Crossbar technology

Bisection width (B_w) = number of nodes (n)

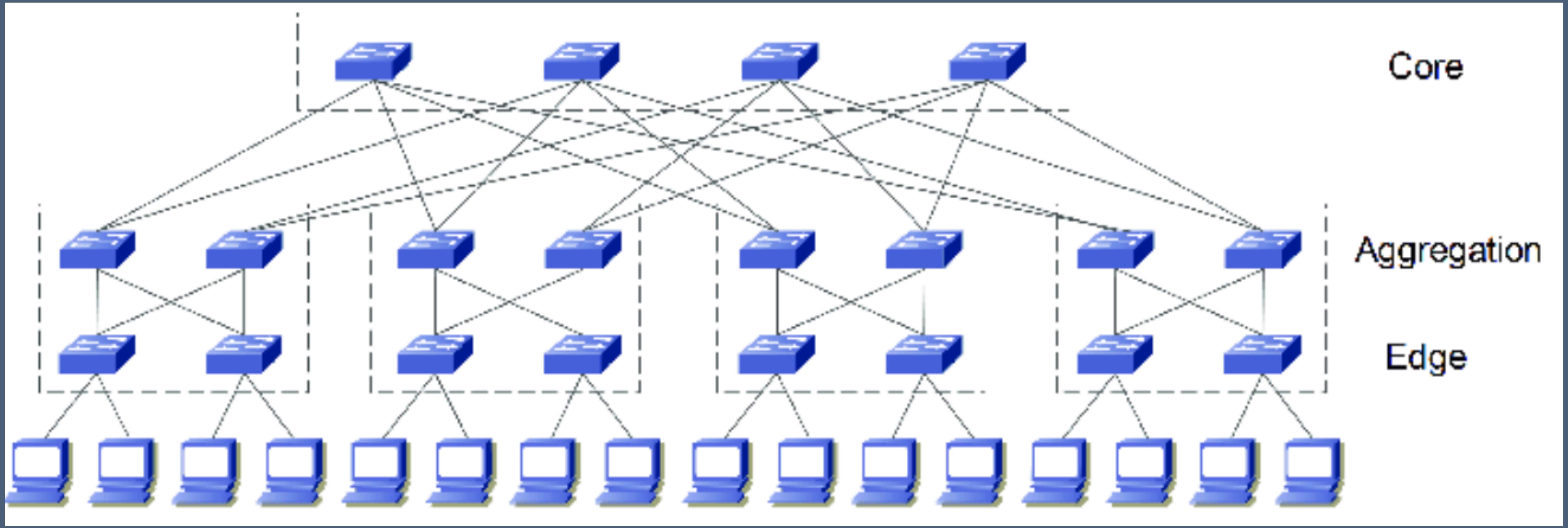
Image Source: https://everythingaboutcomputernetworks.weebly.com/star-topology.html
Image Source: https://www.cs.emory.edu/~cheung/Courses/355/Syllabus/90-parallel/CrossBar.html

Ring Topology



- Typically used in office spaces
- This can provide internet connection even if one connection fails
- $B_w = 2$

Fat Tree Topology

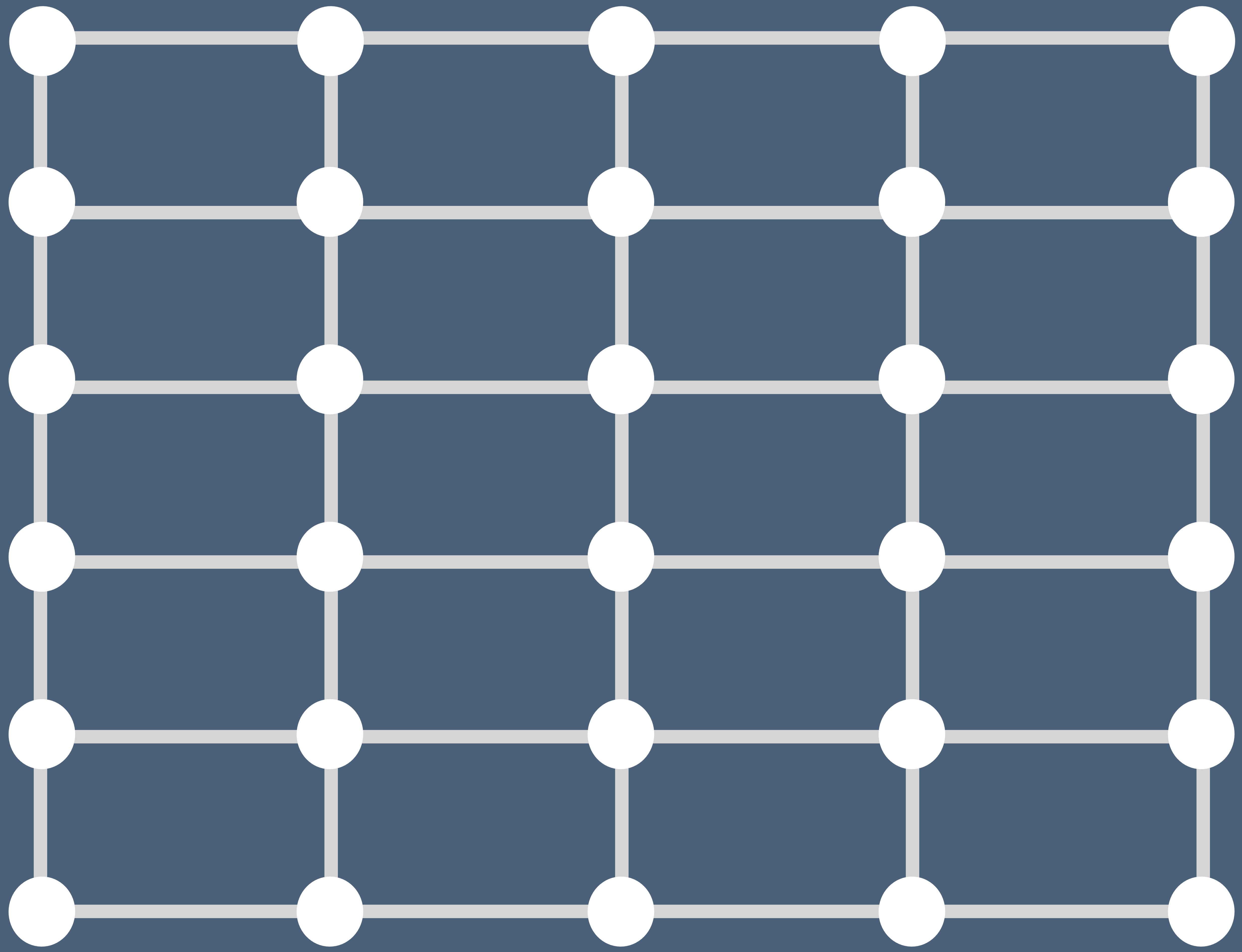


$$B_w = n/2$$

- All switches have same number of ports
- Typically 32/64 port switches are used
- Most high-performance supercomputers on the Top 500 list, including recent leaders like Summit and Sierra, use a fat-tree network due to its high bandwidth and scalability

Image Source: https://networkengineering.stackexchange.com/questions/67729/fat-tree-topology

Mesh Topology



n	B_w
1	0
2	1
4	2
9	3
16	4
25	5

$$B_w = \sqrt{n}$$

3D Torus - Bluegene/P

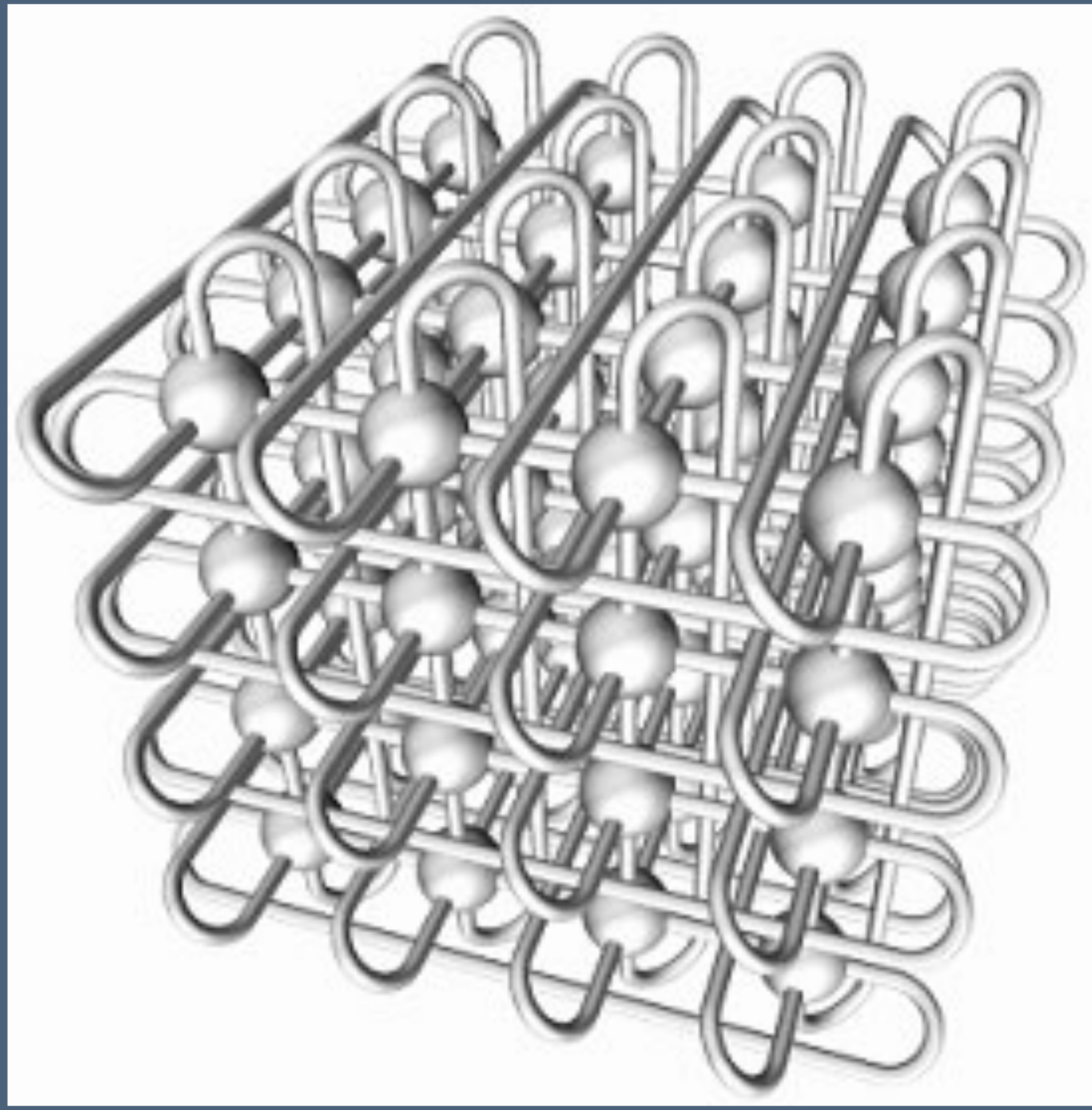


Image Source: https://stackoverflow.com/q/66723243/1525392

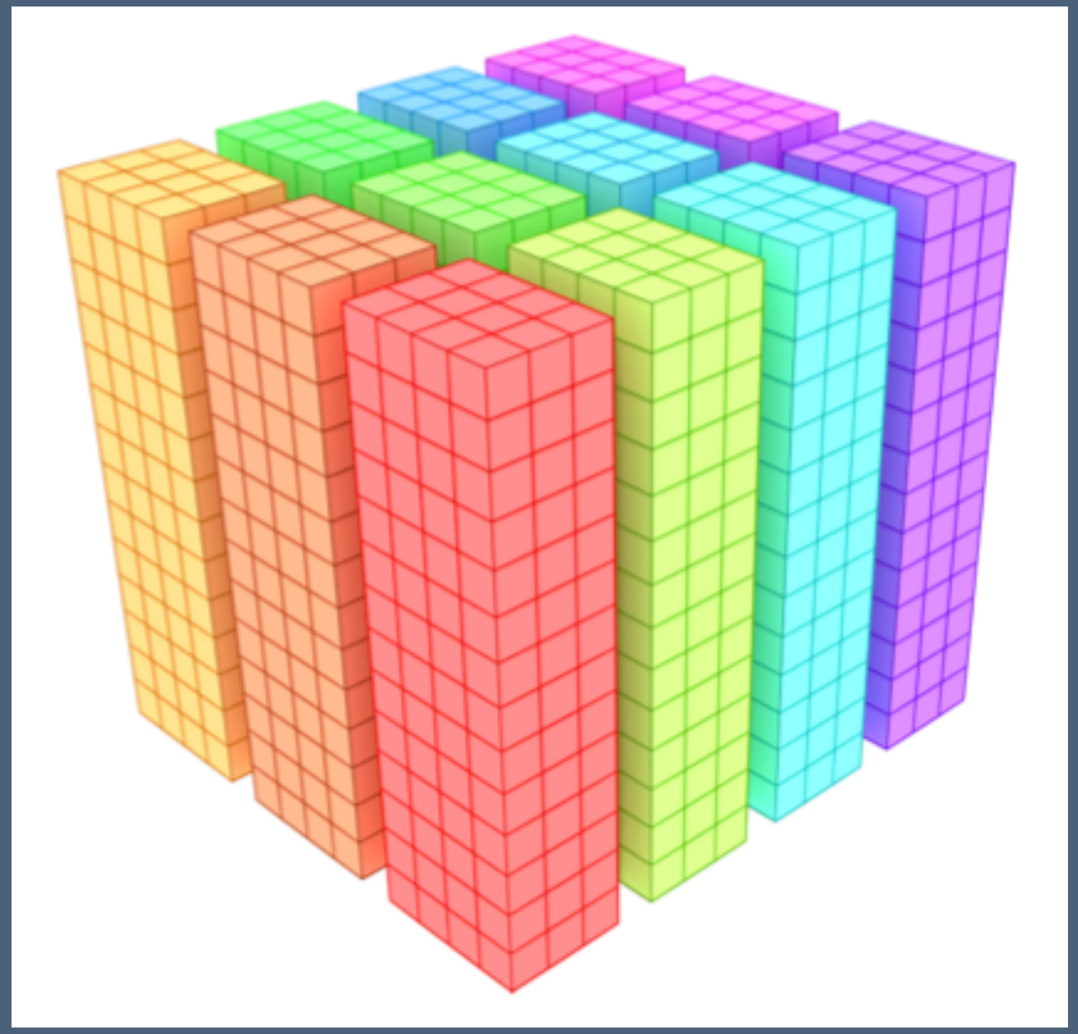
Bisection Bandwidth is proportional to the area

$$B \propto (n')^{2/3}$$

For Pencil Decomposition
square root of n nodes interact ant a time

$$n' = n^{1/2}$$

$$B \propto n^{1/3}$$



A. G. Chatterjee, M. K. Verma, A. Kumar, R. Samtaney, B. Hadri, and R. Khurram, *Scaling of a Fast Fourier Transform and a pseudo-spectral fluid solver up to 196608 cores*, J. Parallel Distrib. Comput.,**113**, 77 (2018)

Pencil Decomposition

Data per node $\propto N^3/n$

Data in wire: $D \propto N^3/n \cdot n^{1/2}$

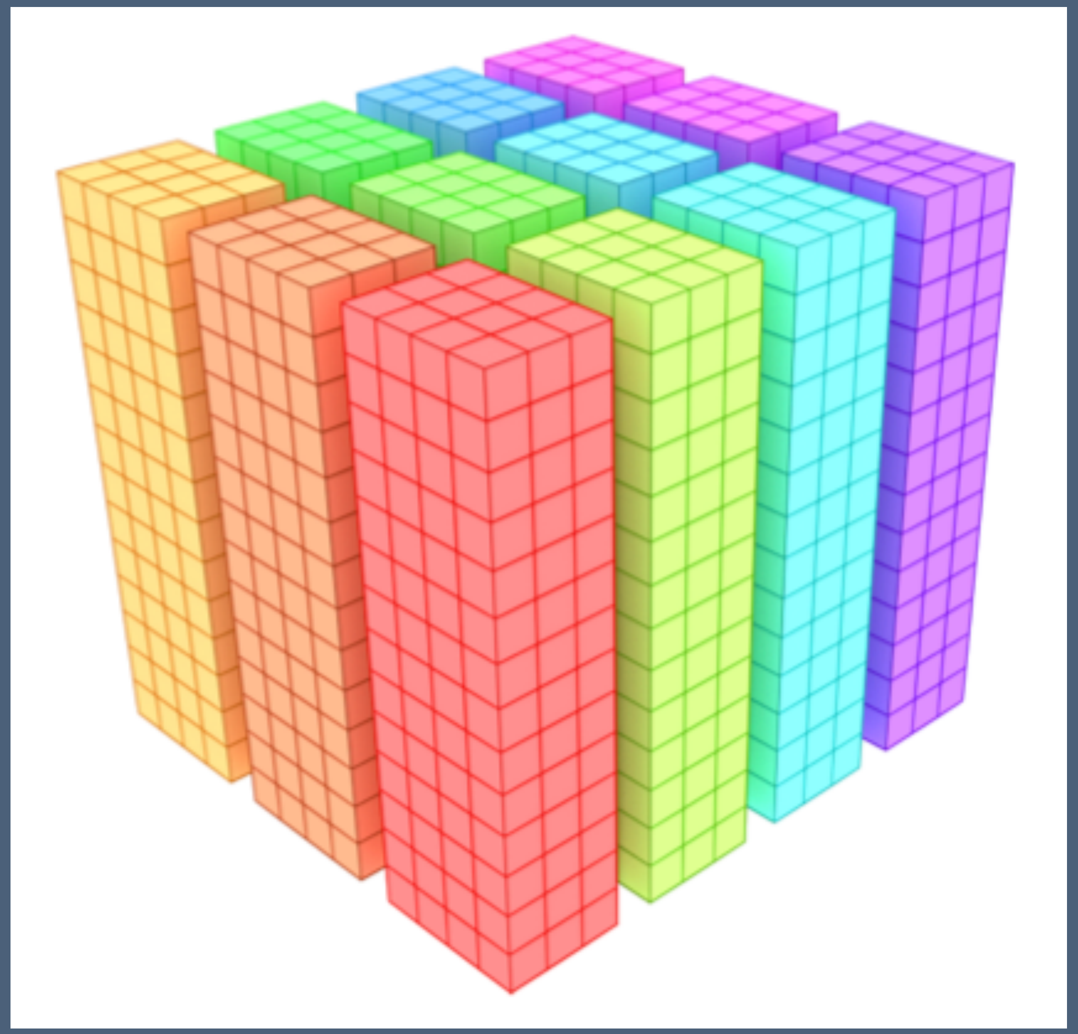
$\propto N^3/n^{1/2}$

By Definition $B = D/T_c$

$$T_c = D/B$$

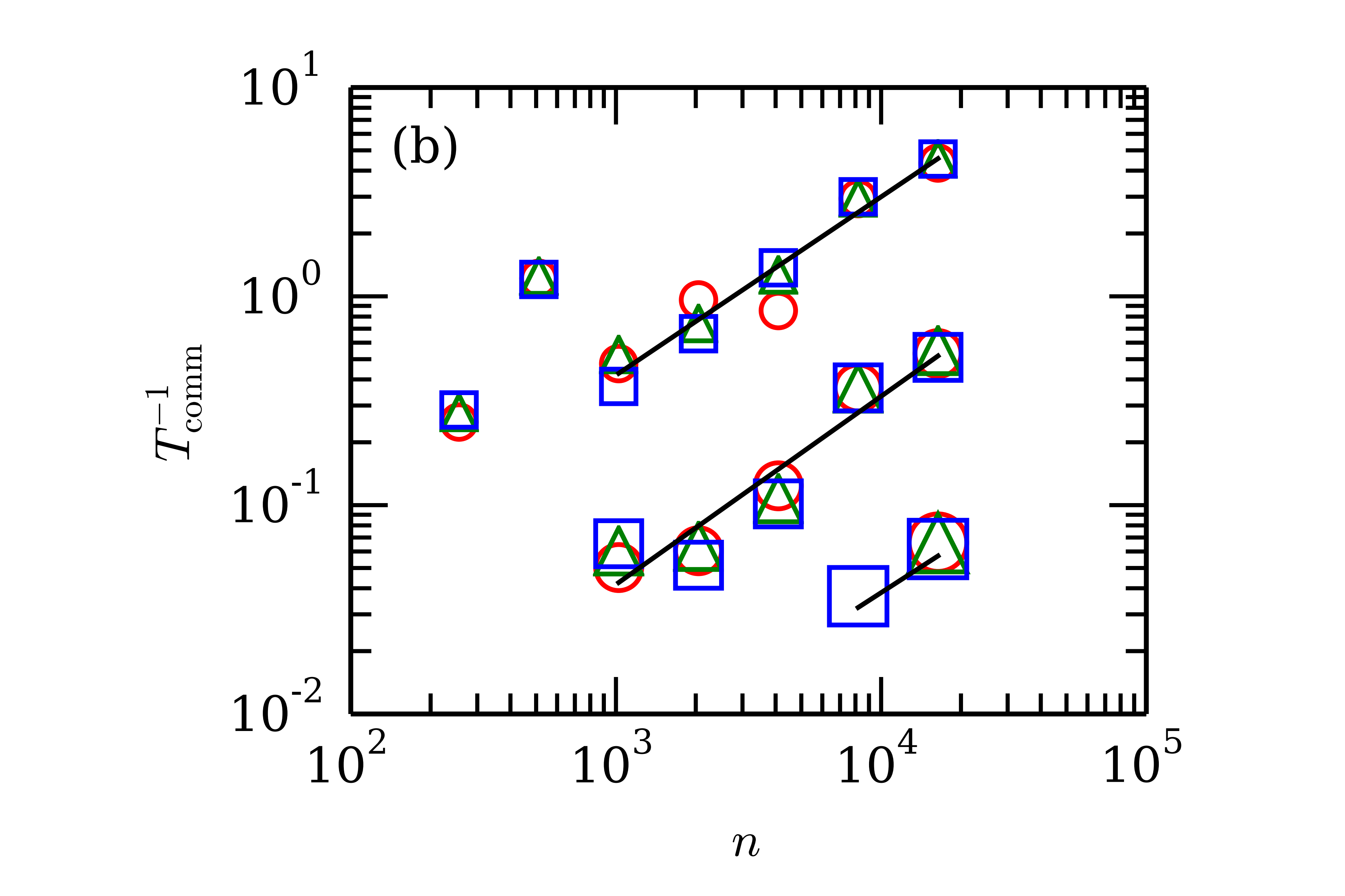
$$= \frac{N^3}{n^{1/2}} \cdot \frac{1}{n^{1/3}} = N^3 \cdot n^{6/5}$$

$$T_c \propto \frac{1}{n^{5/6}} \approx \frac{1}{n^{0.83}}$$



Communication Scaling on Bluegene-P

Shaheen-I, KAUST, SA



2014-2019

~ 65,566 cores

From FFTK Scaling: $T_c \propto \left(\frac{1}{n^{0.8}}\right)$

From Bisection Width Calculations: $T_c \propto \left(\frac{1}{n^{0.83}}\right)$

Best Hardware

Earth Simulator: 2002–2009

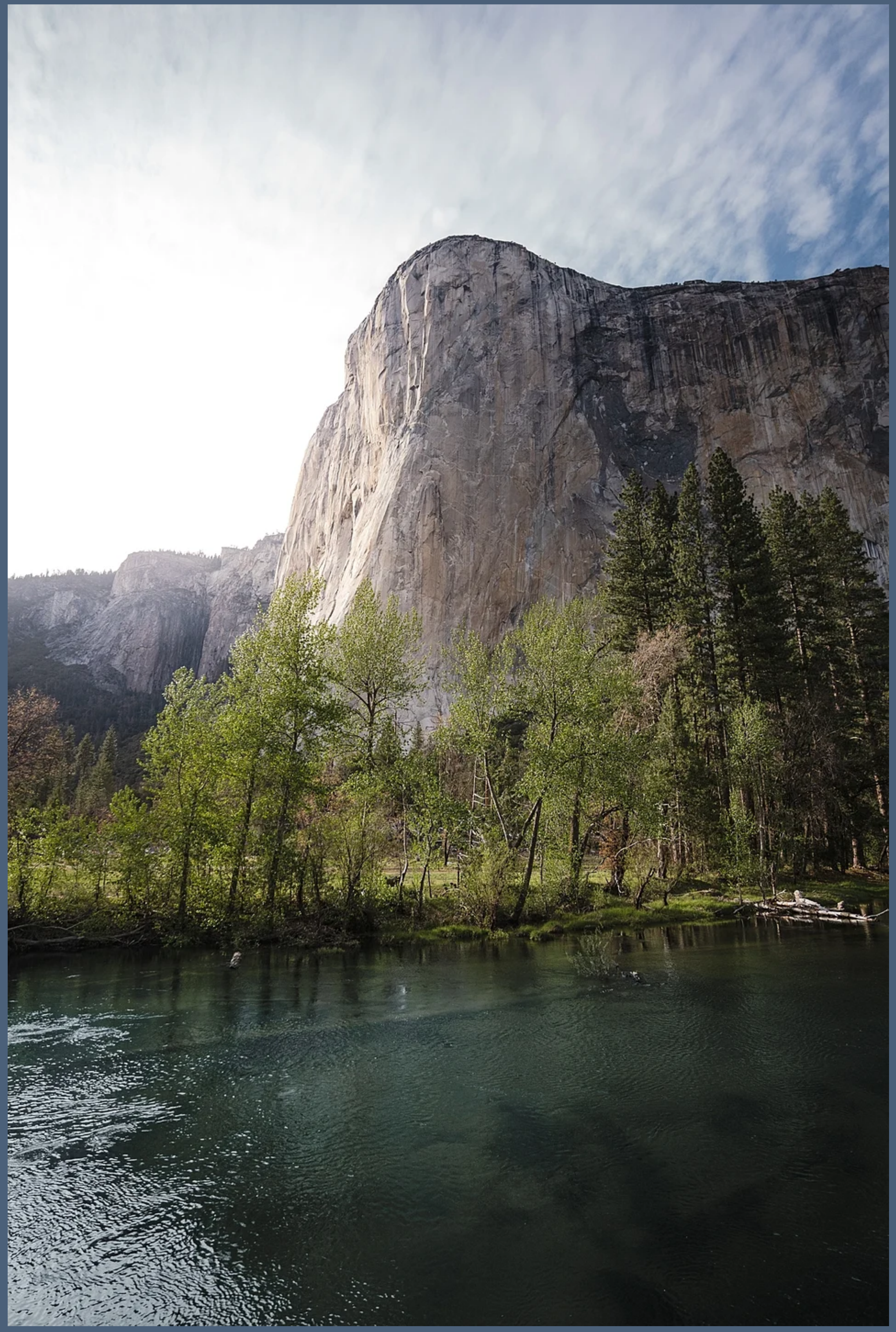
JAMSTEC Yokohama Institute for Earth Sciences

- Earth Simulator Top-ranked the Global FFT at HPC Challenge Awards in 2010
- 640 x 640 single stage crossbar (Star topology)

Press Release: https://www.jamstec.go.jp/e/about/press_release/archive/2010/20101117.pdf

EL CAPITAN

Theoretical Peak (Rpeak)	2,821 PFlop/s
Linpack Performance (Rmax)	1,809 PFlop/s
HPCG	17,406 TFlop/s
Network Bandwidth	1.6 Terra bytes/s

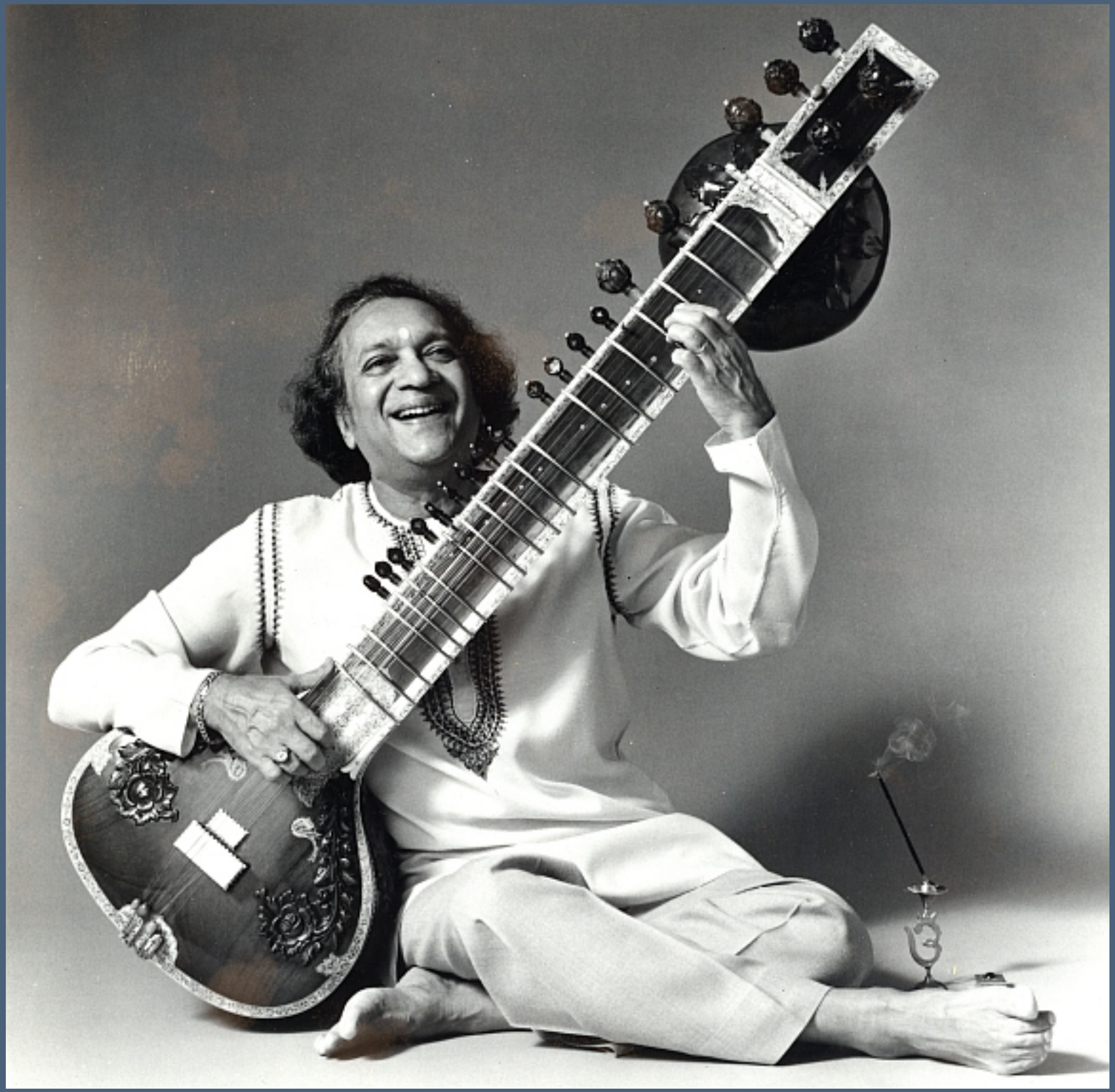


Yosemite National Park
California

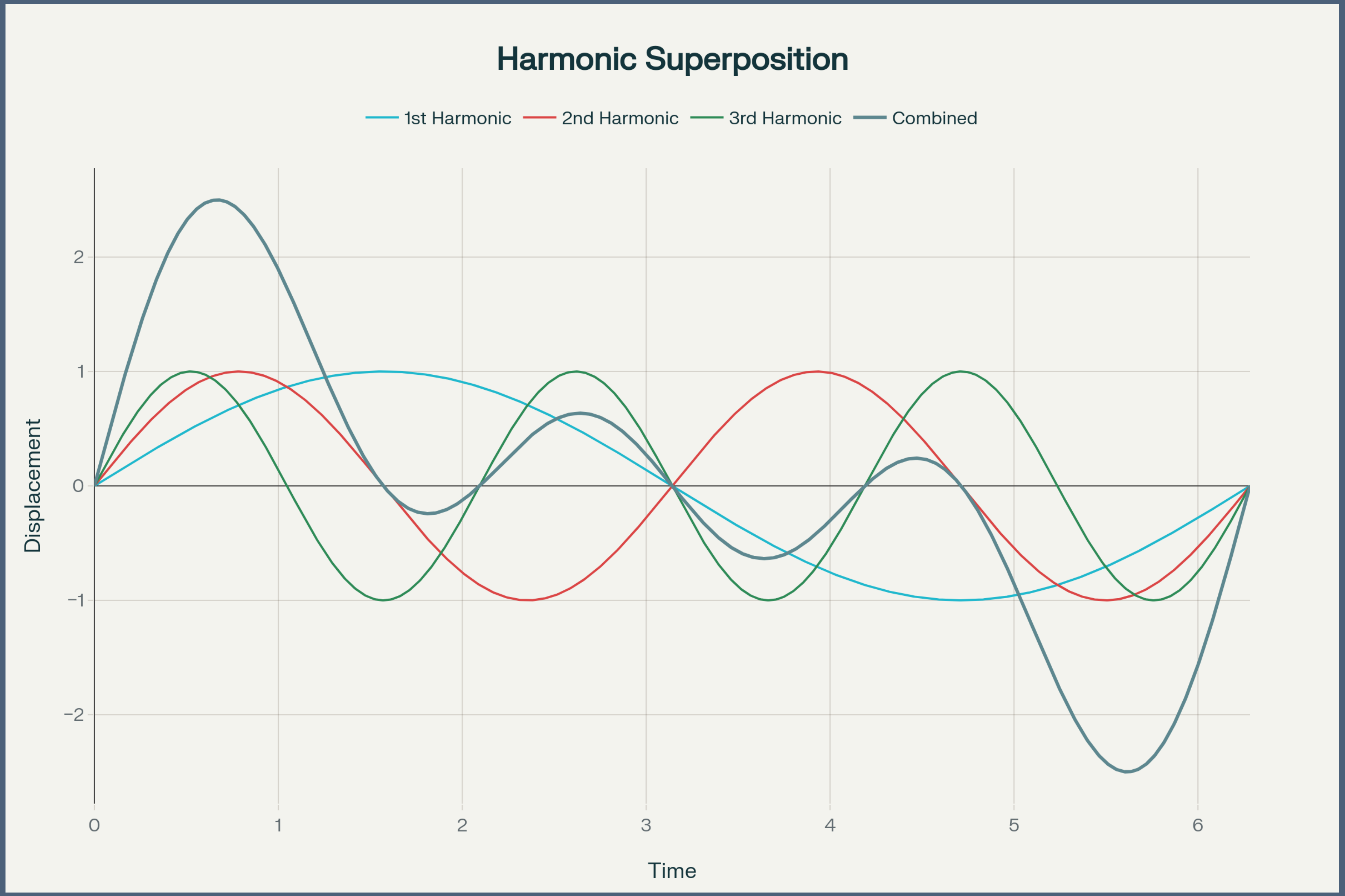
FFT in Nature

Physics of Plucked String Vibrations

Pt Ravi Shankar



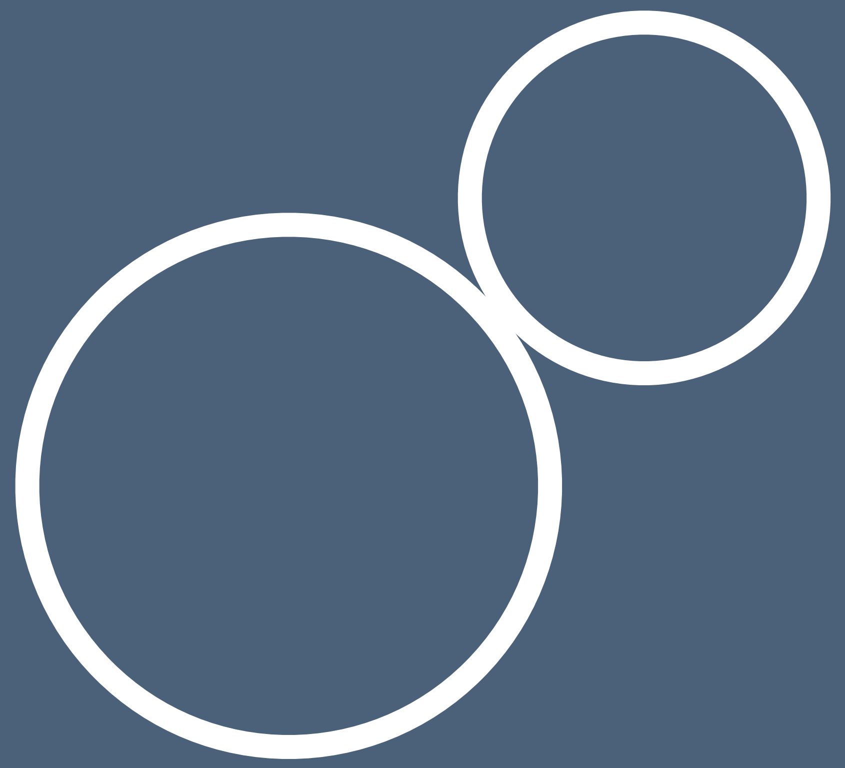
$$f_n = n\frac{1}{2L}\sqrt{\frac{T}{\mu}}$$



Fourier Transform in

Everyday Activities

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\frac{n\pi x}{L}}$$



A Video by: 3Blue1Brown

But what is a Fourier series?
From heat flow to drawing with circles

https://www.youtube.com/watch?v=r6sGWTCMz2k

We can get any pattern by combining infinitely many cycles in appropriate proportions

Thank You