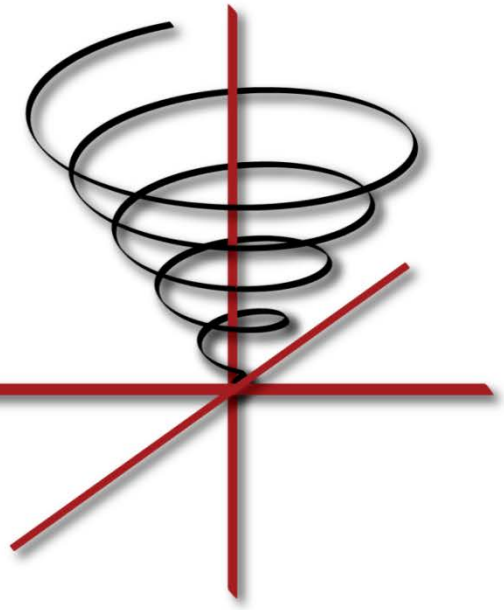
SPIRAL

FFT Library Generation and Autotuning



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In collaboration with the SPIRAL team

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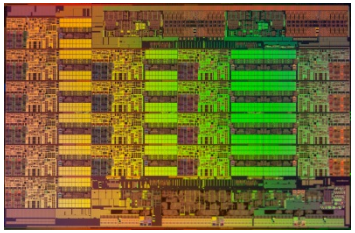
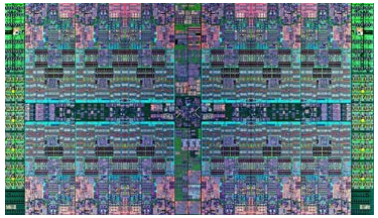
Today's Computing Landscape

1 Gflop/s = one billion floating-point operations (additions or multiplications) per second

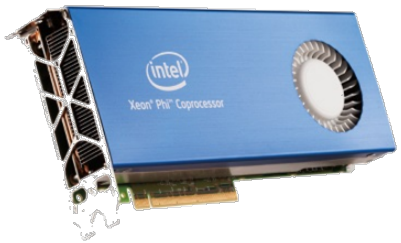


Intel Xeon E5-2699v3
662 Gflop/s, 145 W
18 cores, 2.3 GHz
4-way/8-way AVX2



IBM POWER8
384 Gflop/s, 200 W
12 cores, 4 GHz
2-way/4-way VMX/VSX

Nvidia Tesla M2090
666 Gflop/s, 225 W
512 cores, 1.33 GHz
32-way SIMT



Intel Xeon Phi
1.2 Tflop/s, 300 W
61 cores, 1.24 GHz
8-way/16-way LRBni

Snapdragon 810
10 Gflop/s, 2 W
4 cores, 2.5 GHz
A330 GPU, V50 DSP, NEON

Intel Atom C2750
29 Gflop/s, 20 W
8 cores, 2.4 GHz
2-way/4-way SSSE3

Dell PowerEdge R920
1.34 Tflop/s, 850 W
4x 15 cores, 2.8 GHz
4-way/8-way AVX

BlueGene/Q
10 Pflop/s, 8 MW
48k x 16 cores, 1.6 GHz
4-way QPX

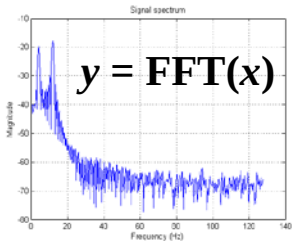
Idea: Go from Mathematics to Software

Given:

- **Mathematical problem specification**
  does not change
- **Computer platform**
  changes often

Wanted:

- **Very good implementation of specification on platform**
- **Proof of correctness**



on

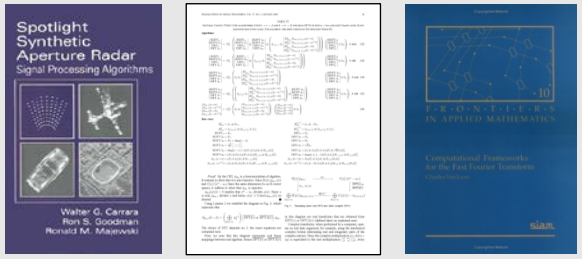
automatic

```
void fft64(double  *Y, double  *X) {
    ...
    s5674 = _mm256_permute2f128_pd(s5672, s5673, (0) | ((2) << 4));
    s5675 = _mm256_permute2f128_pd(s5672, s5673, (1) | ((3) << 4));
    s5676 = _mm256_unpacklo_pd(s5674, s5675);
    s5677 = _mm256_unpackhi_pd(s5674, s5675);
    s5678 = *((a3738 + 16));
    s5679 = *((a3738 + 17));
    s5680 = _mm256_permute2f128_pd(s5678, s5679, (0) | ((2) << 4));
    s5681 = _mm256_permute2f128_pd(s5678, s5679, (1) | ((3) << 4));
    s5682 = _mm256_unpacklo_pd(s5680, s5681);
    s5683 = _mm256_unpackhi_pd(s5680, s5681);
    t5735 = _mm256_add_pd(s5676, s5682);
    t5736 = _mm256_add_pd(s5677, s5683);
    t5737 = _mm256_add_pd(s5670, t5735);
    t5738 = _mm256_add_pd(s5671, t5736);
    t5739 = _mm256_sub_pd(s5670, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5735));
    t5740 = _mm256_sub_pd(s5671, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5736));
    t5741 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5677, s5683));
    t5742 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5676, s5682));
    ...
}
```

QED.

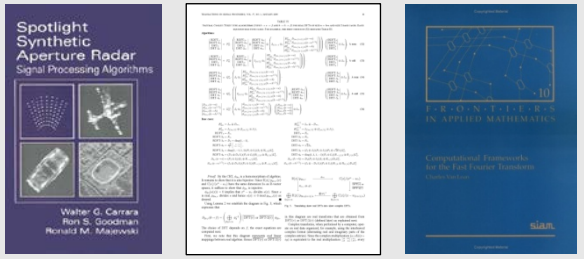
What is Spiral?

Traditionally



High performance library optimized for given platform

Spiral Approach



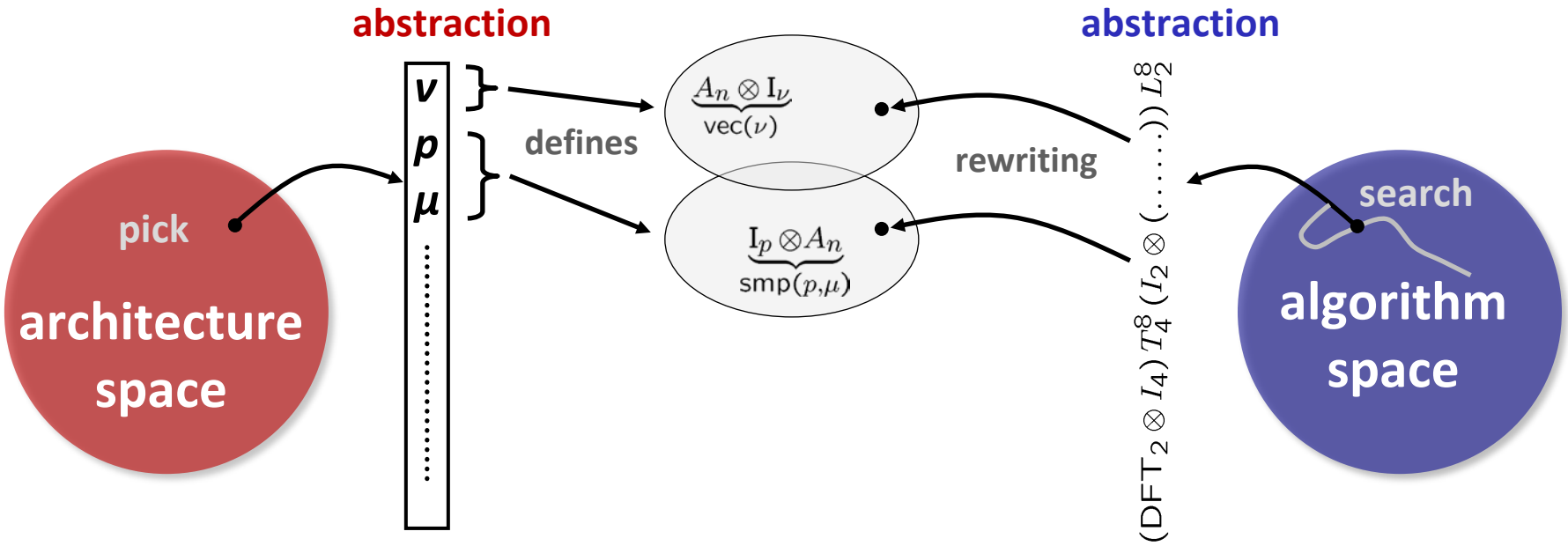
Spiral

High performance library optimized for given platform

Comparable performance

Platform-Aware Formal Program Synthesis

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

optimization

Kernel:
problem size,
algorithm choice

Algorithms: Rules in Domain Specific Language

Linear Transforms

$$
\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m)Q_n, \quad n = km,\ \gcd(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})D_p(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\, \mathrm{L}^n_m(\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\quad \cdot(\mathrm{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m)\left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right)\right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathrm{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathrm{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathrm{J}_2\, \mathrm{R}_{13\pi/8}
\end{aligned}
$$

Viterbi Decoding

$$
\begin{aligned}
\underbrace{\mathbf{Vit}}_{\mathrm{vec}(v)} &\rightarrow \underbrace{\left(\prod (L \times I) \circ (I \otimes C)\right) \circ Id}_{\mathrm{vec}(v)} \\
&\rightarrow \left(\prod \underbrace{(L \times I) \circ (I \otimes C)}_{\mathrm{vec}(v)}\right) \circ Id \\
\times \quad &\rightarrow \left(\prod (L \otimes I_v \times I) \circ (I \otimes C \otimes I_v) \circ (\vec{L} \times I)\right) \circ Id \\
&\rightarrow \prod (L \otimes I_v \times I) \circ (I \otimes (B \otimes I_v)) \circ (\vec{L} \times I)
\end{aligned}
$$

Matrix-Matrix Multiplication

$$
\begin{aligned}
&\mathrm{MMM}_{1,1,1} \rightarrow (\cdot)_1 \\
&\mathrm{MMM}_{m,n,k} \rightarrow (\otimes)_{m/m_b \times 1} \otimes \mathrm{MMM}_{m_b,n,k} \\
&\mathrm{MMM}_{m,n,k} \rightarrow \mathrm{MMM}_{m,nb,k} \otimes (\otimes)_{1 \times n/nb} \\
&\mathrm{MMM}_{m,n,k} \rightarrow ((\Sigma_{k/k_b} \circ (\cdot)_{k/k_b}) \otimes \mathrm{MMM}_{m,n,k_b}) \circ \\
&\qquad\qquad ((L^{mk/k_b}_{k/k_b} \otimes I_{k_b}) \times I_{kn}) \\
&\mathrm{MMM}_{m,n,k} \rightarrow (L^{mn/n_b}_m \otimes I_{n_b}) \circ \\
&\qquad ((\otimes)_{1 \times n/n_b} \otimes \mathrm{MMM}_{m,n_b,k}) \circ \\
&\qquad\qquad (I_{km} \times (L^{kn/n_b}_{n/n_b} \otimes I_{n_b}))
\end{aligned}
$$

Synthetic Aperture Radar (SAR)

$$
\begin{aligned}
\mathrm{SAR}_{k \times m \rightarrow n \times n} &\rightarrow \mathrm{DFT}_{n \times n} \circ \mathrm{Interp}_{k \times m \rightarrow n \times n} \\
\mathrm{DFT}_{n \times n} &\rightarrow (\mathrm{DFT}_n \otimes \mathrm{I}_n) \circ (\mathrm{I}_n \otimes \mathrm{DFT}_n) \\
\mathrm{Interp}_{k \times m \rightarrow n \times n} &\rightarrow (\mathrm{Interp}_{k \rightarrow n} \otimes_i \mathrm{I}_n) \circ (\mathrm{I}_k \otimes_i \mathrm{Interp}_{m \rightarrow n}) \\
\mathrm{Interp}_{r \rightarrow s} &\rightarrow \left(\bigoplus_{i=0}^{n-2} \mathrm{InterpSeg}_k\right) \oplus \mathrm{InterpSegPruned}_{k,\ell} \\
\mathrm{InterpSeg}_k &\rightarrow \mathrm{G}^{u \cdot n \rightarrow k}_f \circ \mathrm{iPrunedDFT}_{n \rightarrow u \cdot n} \circ \left(\frac{1}{n}\right) \circ \mathrm{DFT}_n
\end{aligned}
$$

Formal Approach for all Types of Parallelism

- **Multithreading** (Multicore)
- **Vector SIMD** (SSE, VMX/Altivec,…)
- **Message Passing** (Clusters, MPP)
- **Streaming/multibuffering** (Cell)
- **Graphics Processors** (GPUs)
- **Gate-level parallelism** (FPGA)
- **HW/SW partitioning** (CPU + FPGA)

$$\mathrm{I}_p \otimes_{\parallel} A_{\mu n}, \quad \mathbf{L}^{mn}_{m} \bar{\otimes} \mathrm{I}_{\mu}$$

$$A \hat{\otimes} \mathrm{I}_{\nu} \quad \underbrace{\mathbf{L}^{2\nu}_{2}}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{2\nu}_{\nu}}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{\nu^2}_{\nu}}_{\text{isa}}$$

$$\mathrm{I}_p \otimes_{\parallel} A_n, \quad \underbrace{\mathbf{L}^{p^2}_{p} \bar{\otimes} \mathrm{I}_{n/p^2}}_{\text{all-to-all}}$$

$$\mathrm{I}_n \otimes_2 A_{\mu n}, \quad \mathbf{L}^{mn}_{m} \bar{\otimes} \mathrm{I}_{\mu}$$

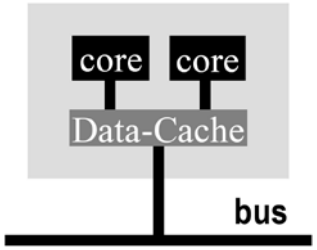
$$\prod_{i=0}^{n-1} A_i, \quad A_n \hat{\otimes} \mathrm{I}_w, \quad P_n \otimes Q_w$$

$$\prod_{i=0}^{n-1^{\mathrm{ir}}} A, \quad \mathrm{I}_s \tilde{\otimes} A, \quad \underbrace{\mathbf{L}^{m}_{n}}_{\text{bram}}$$

$$\underbrace{A_1}_{\text{fpga}}, \quad \underbrace{A_2}_{\text{fpga}}, \quad \underbrace{A_3}_{\text{fpga}}, \quad \underbrace{A_4}_{\text{fpga}}$$

Modeling Hardware: Base Cases

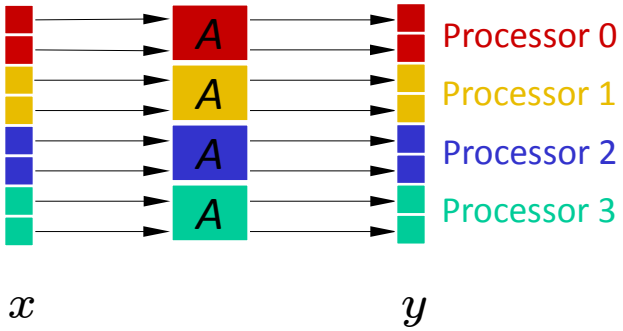
- **Hardware abstraction: shared cache with cache lines**



$$\underbrace{A_{k\times m\to n}}_{\mathrm{smp}(p,\mu)}$$

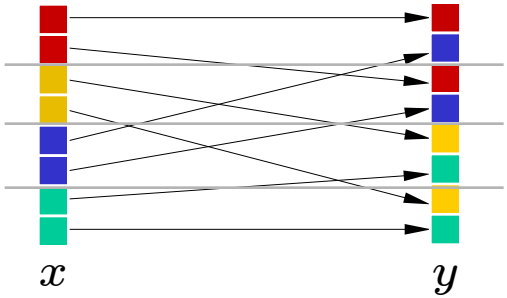
- **Tensor product: embarrassingly parallel operator**

$$y = \left(\mathrm{I}_p \otimes A\right)(x)$$



- **Permutation: problematic; may produce false sharing**

$$y = \mathrm{L}^{8}_{4}(x)$$



Example Program Transformation Rule Set

$$\underbrace{AB}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{A}_{\mathsf{smp}(p,\mu)}\ \underbrace{B}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{A_m \otimes \mathrm{I}_n}_{\mathsf{smp}(p,\mu)} \rightarrow \underbrace{\left(\mathrm{L}^{mp}_{m} \otimes \mathrm{I}_{n/p}\right)\left(\mathrm{I}_p \otimes (A_m \otimes \mathrm{I}_{n/p})\right)\left(\mathrm{L}^{mp}_{p} \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)}$$

$$\underbrace{\mathrm{L}^{mn}_{m}}_{\mathsf{smp}(p,\mu)} \rightarrow \begin{cases} \underbrace{\left(\mathrm{I}_p \otimes \mathrm{L}^{mn/p}_{m/p}\right)}_{\mathsf{smp}(p,\mu)}\underbrace{\left(\mathrm{L}^{pn}_{p} \otimes \mathrm{I}_{m/p}\right)}_{\mathsf{smp}(p,\mu)} \\ \underbrace{\left(\mathrm{L}^{pm}_{m} \otimes \mathrm{I}_{n/p}\right)}_{\mathsf{smp}(p,\mu)}\underbrace{\left(\mathrm{I}_p \otimes \mathrm{L}^{mn/p}_{m}\right)}_{\mathsf{smp}(p,\mu)} \end{cases}$$

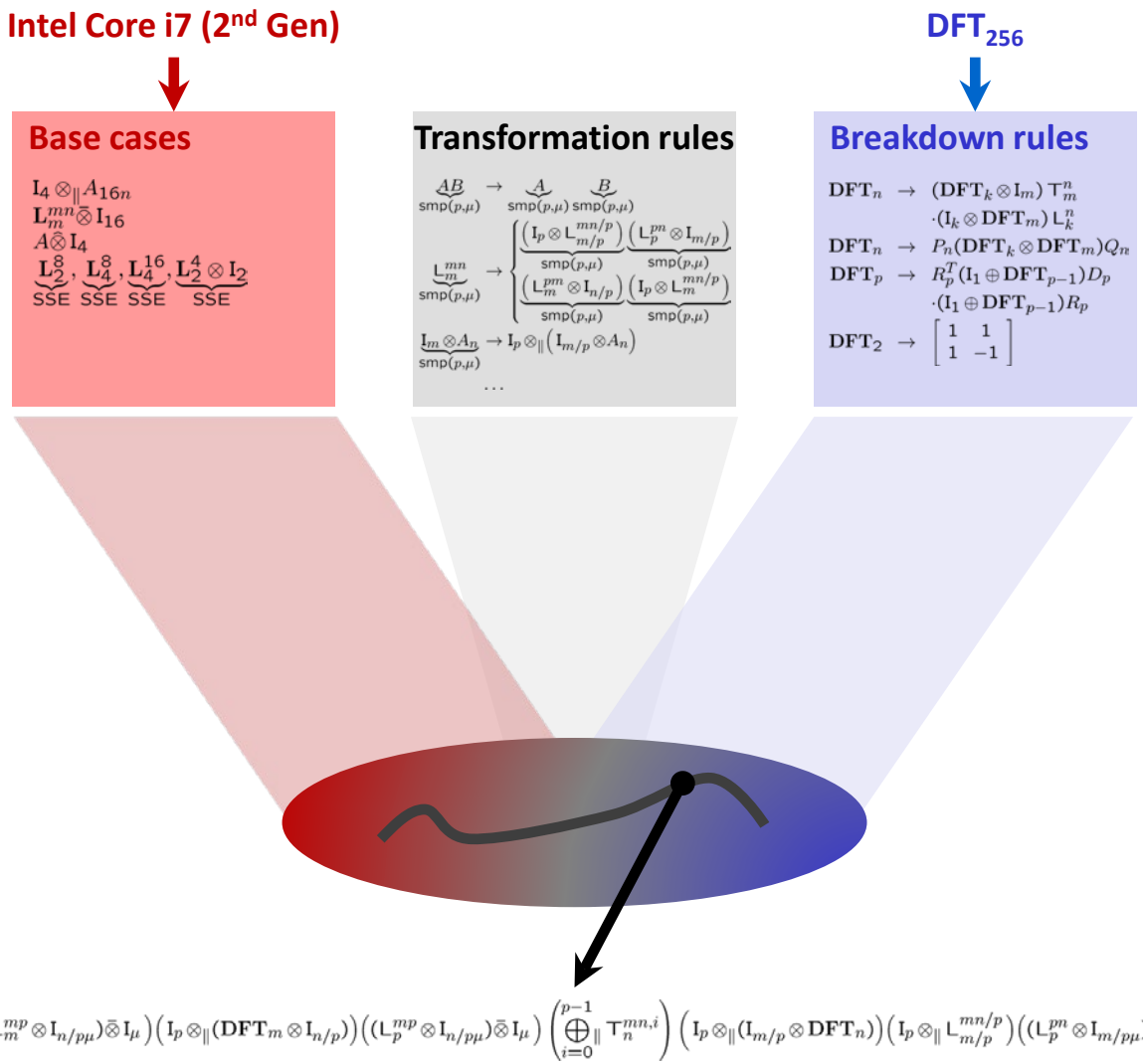
Recursive rules

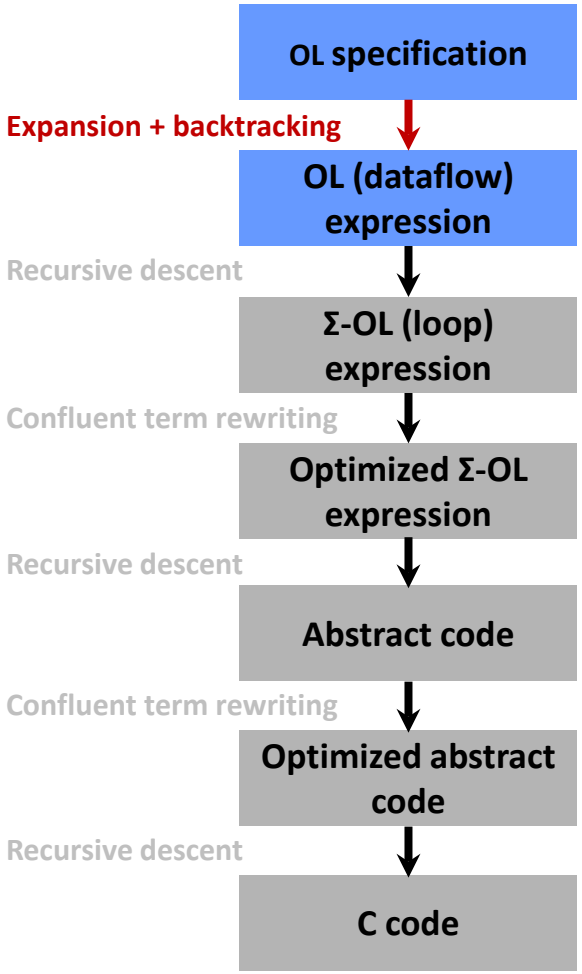
$$\underbrace{\mathrm{I}_m \otimes A_n}_{\mathsf{smp}(p,\mu)} \rightarrow \mathrm{I}_p \otimes_{\|} \left(\mathrm{I}_{m/p} \otimes A_n\right)$$

$$\underbrace{(P \otimes \mathrm{I}_n)}_{\mathsf{smp}(p,\mu)} \rightarrow \left(P \otimes \mathrm{I}_{n/\mu}\right) \overline{\otimes}\, \mathrm{I}_\mu$$

Base case rules

Autotuning in Constraint Solution Space





Translating an OL Expression Into Code

Constraint Solver Input: $\underbrace{\mathrm{DFT}_8}_{\text{double}}$

Output =

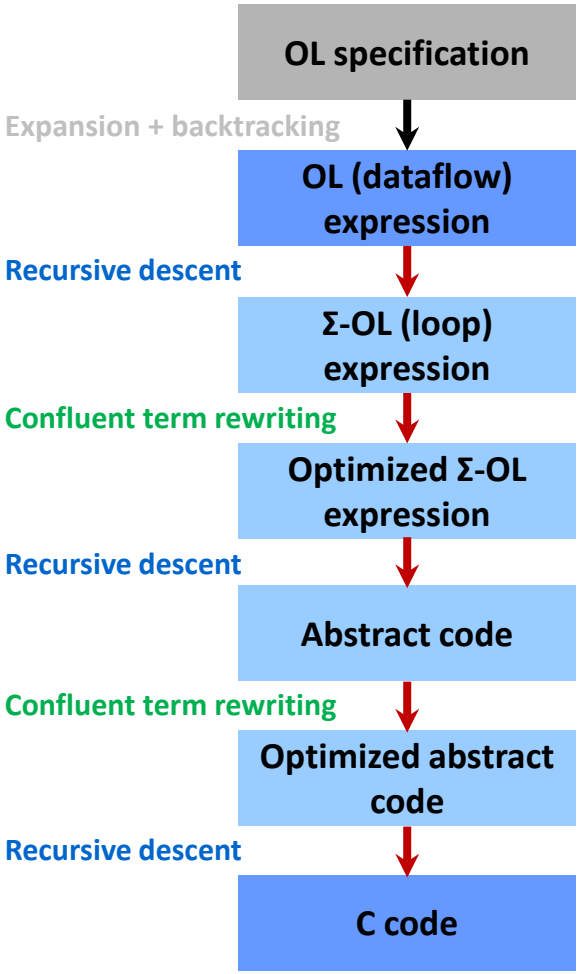
OL Expression: $(\mathrm{DFT}_2 \otimes I_4)\, T^8_4 \left(I_2 \otimes \left((\mathrm{DFT}_2 \otimes I_2)\, T^4_2\, (I_2 \otimes \mathrm{DFT}_2)\, L^4_2\right)\right) L^8_2$

Σ-OL:

$$\sum_{j=0}^{3} \left(S_j\, \mathrm{DFT}_2\, G_j\right) \sum_{k=0}^{1} \left(\sum_{l=0}^{1} \left(S_{k,l}\, \mathrm{diag}\left(\mathrm{t}_{k,l}\right) \mathrm{DFT}_2\, G_l\right) \sum_{m=0}^{1} \left(S_m\, \mathrm{diag}\left(\mathrm{t}_m\right) \mathrm{DFT}_2\, G_{k,m}\right)\right)$$

C Code:

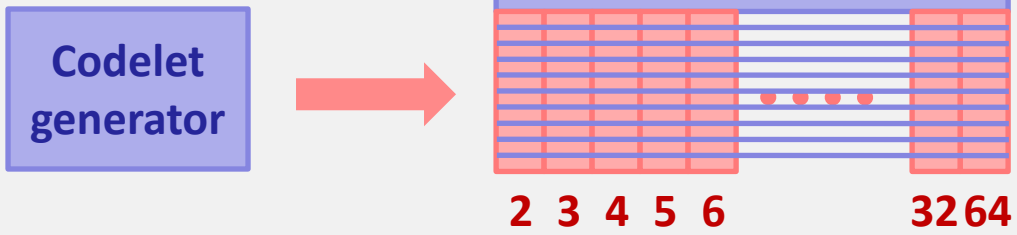
```
void sub(double *y, double *x) {
  double f0, f1, f2, f3, f4, f7, f8, f10, f11;
  f0 = x[0] - x[3];
  f1 = x[0] + x[3];
  f2 = x[1] - x[2];
  f3 = x[1] + x[2];
  f4 = f1 - f3;
  y[0] = f1 + f3;
  y[2] = 0.7071067811865476 * f4;
  f7 = 0.9238795325112867 * f0;
  f8 = 0.3826834323650898 * f2;
  y[1] = f7 + f8;
  f10 = 0.3826834323650898 * f0;
  f11 = (-0.9238795325112867) * f2;
  y[3] = f10 + f11;
}
```



SPIRAL: Meta-Tool to FFTW

blue = hand-written, red = generated

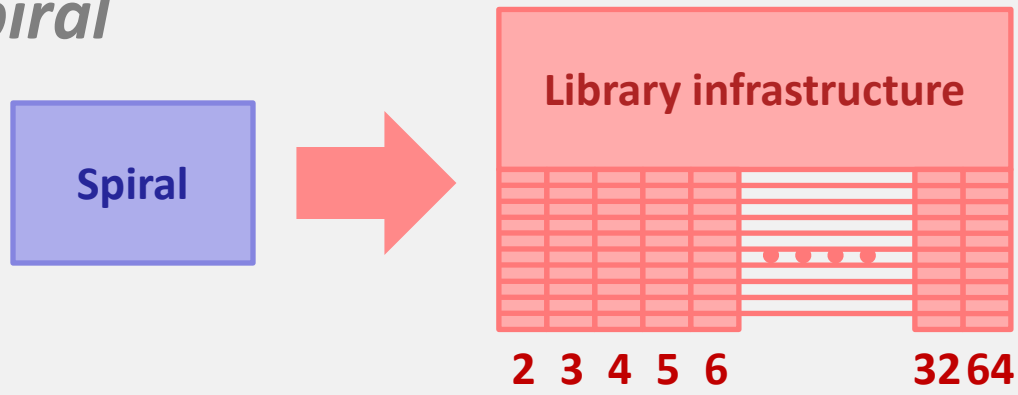
FFTW



Recursive functions needed
Vectorization (partially)
Threading
Adaptation mechanism (plan)

Codelets (10–20 types)

Spiral



Recursive functions needed
Vectorization
Threading
Adaptation mechanism (plan)

Codelets (10–20 types)

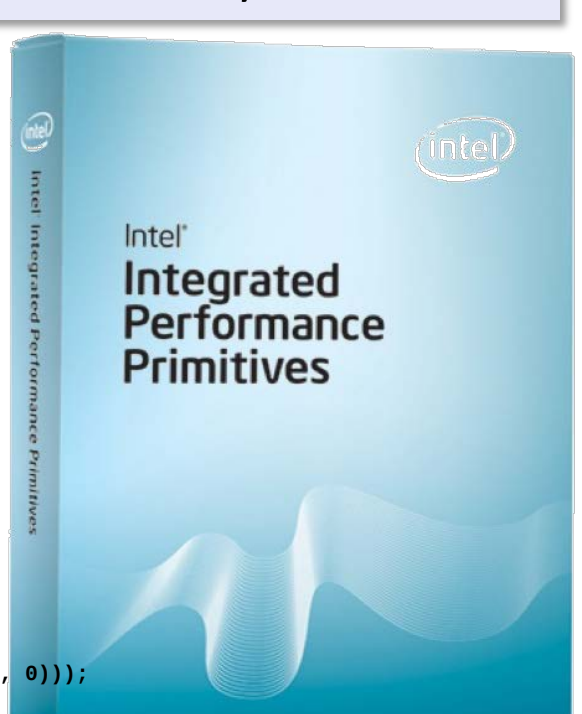
Production-Quality Code

```
...
s3013 = _mm_loadl_pi(a772, ((float  *) X));
s3014 = _mm_loadh_pi(_mm_loadl_pi(a772, ((float  *) (X + 2))), ((float  *) (X + 6)));
```

Spiral-Synthesized code in Intel's Library IPP 6 and 7

- IPP = Intel's performance primitives, part of Intel C++ Compiler suite
- Generated: 3984 C functions (signal processing) = 1M lines of code
- Full parallelism support
- Computer-generated code: Faster than what was achievable by hand

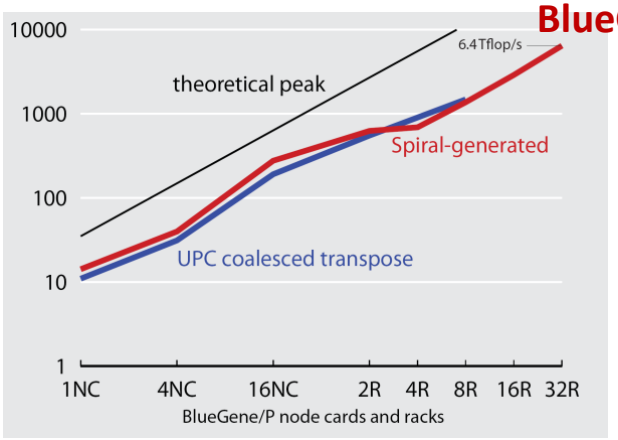
```
s3017 = _mm_loadl_pi(a772, ((float  *) (X + 14)));
s3018 = _mm_loadh_pi(_mm_loadl_pi(a772, ((float  *) (X + 24))), ((float  *) (X + 20)));
s3019 = _mm_loadl_pi(a772, ((float  *) (X + 8)));
s3020 = _mm_loadh_pi(_mm_loadl_pi(a772, ((float  *) (X + 10))), ((float  *) (X + 4)));
s3021 = _mm_loadl_pi(a772, ((float  *) (X + 12)));
s3022 = _mm_shuffle_ps(s3014, s3015, _MM_SHUFFLE(2, 0, 2, 0));
s3023 = _mm_shuffle_ps(s3014, s3015, _MM_SHUFFLE(3, 1, 3, 1));
s3024 = _mm_shuffle_ps(s3016, s3017, _MM_SHUFFLE(2, 0, 2, 0));
s3025 = _mm_shuffle_ps(s3016, s3017, _MM_SHUFFLE(3, 1, 3, 1));
s3026 = _mm_shuffle_ps(s3018, s3019, _MM_SHUFFLE(2, 0, 2, 0));
s3027 = _mm_shuffle_ps(s3018, s3019, _MM_SHUFFLE(3, 1, 3, 1));
s3028 = _mm_shuffle_ps(s3020, s3021, _MM_SHUFFLE(2, 0, 2, 0));
s3029 = _mm_shuffle_ps(s3020, s3021, _MM_SHUFFLE(3, 1, 3, 1));
...
t3794 = _mm_add_ps(s3042, s3043);
t3795 = _mm_add_ps(s3038, t3793);
t3796 = _mm_add_ps(s3041, t3794);
t3797 = _mm_sub_ps(s3038, _mm_mul_ps(_mm_set1_ps(0.5), t3793));
t3798 = _mm_sub_ps(s3041, _mm_mul_ps(_mm_set1_ps(0.5), t3794));
s3044 = _mm_mul_ps(_mm_set1_ps(0.8660254037844386), _mm_sub_ps(s3042, s3043));
s3045 = _mm_mul_ps(_mm_set1_ps(0.8660254037844386), _mm_sub_ps(s3039, s3040));
t3799 = _mm_add_ps(t3797, s3044);
t3800 = _mm_sub_ps(t3798, s3045);
t3801 = _mm_sub_ps(t3797, s3044);
t3802 = _mm_add_ps(t3798, s3045);
a773 = _mm_mul_ps(_mm_set_ps(0, 0, 0, 1), _mm_shuffle_ps(s3013, a772, _MM_SHUFFLE(2, 0, 2, 0)));
t3803 = _mm_add_ps(a773, _mm_mul_ps(_mm_set_ps(0, 0, 0, 1), t3795));
a774 = _mm_mul_ps(_mm_set_ps(0, 0, 0, 1), _mm_shuffle_ps(s3013, a772, _MM_SHUFFLE(3, 1, 3, 1)));
t3804 = _mm_add_ps(a774, _mm_mul_ps(_mm_set_ps(0, 0, 0, 1), t3796));
t3805 = _mm_add_ps(a773, _mm_add_ps(_mm_mul_ps(_mm_set_ps(0.28757036473700154, 0.30046260628866578, (-0.28757036473700154), (-0.0833333333333333
t3806 = _mm_add_ps(a774, _mm_add_ps(_mm_mul_ps(_mm_set_ps(0.08706930057606789, 0, 0.087069300576068001, 0), t3795), _mm_mul_ps(_mm_set_ps(0.2875
s3046 = _mm_sub_ps(_mm_mul_ps(_mm_set_ps((-0.25624767158293649), 0.25826039031174486, (-0.30023863596633249), 0.075902986037193879), t3799), _m
s3047 = _mm_add_ps(_mm_mul_ps(_mm_set_ps(0.15689139105158462, (-0.15355568557954136), (-0.011599105605768193), 0.29071724147084099), t3799), _m
```



SPIRAL: Success in HPC/Supercomputing

- **NCSA Blue Waters**
  PAID Program, FFTs for Blue Waters
- **RIKEN K computer**
  FFTs for the HPC-ACE ISA
- **LANL RoadRunner**
  FFTs for the Cell processor
- **PSC/XSEDE Bridges**
  Large size FFTs
- **LLNL BlueGene/L and P**
  FFTW for BlueGene/L's Double FPU
- **ANL BlueGene/Q Mira**
  Early Science Program, FFTW for BGQ QPX

Global FFT (1D FFT, HPC Challenge)



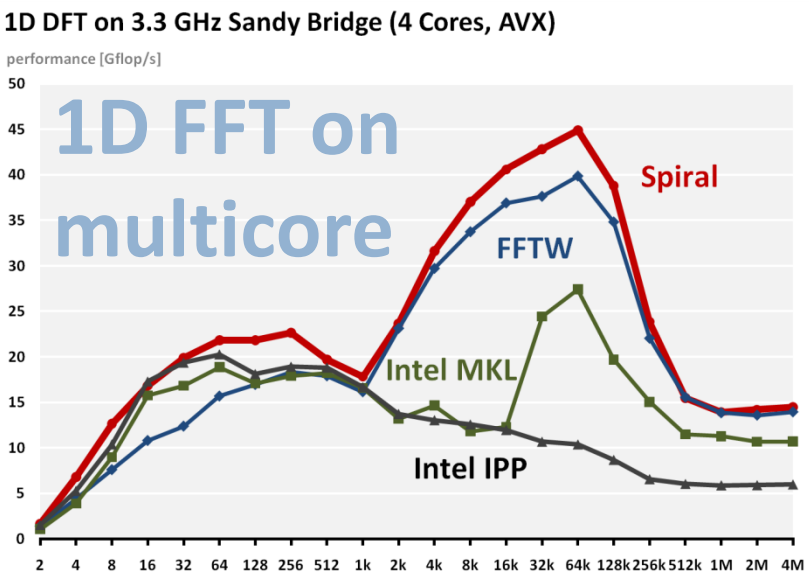
6.4 Tflop/s on BlueGene/P

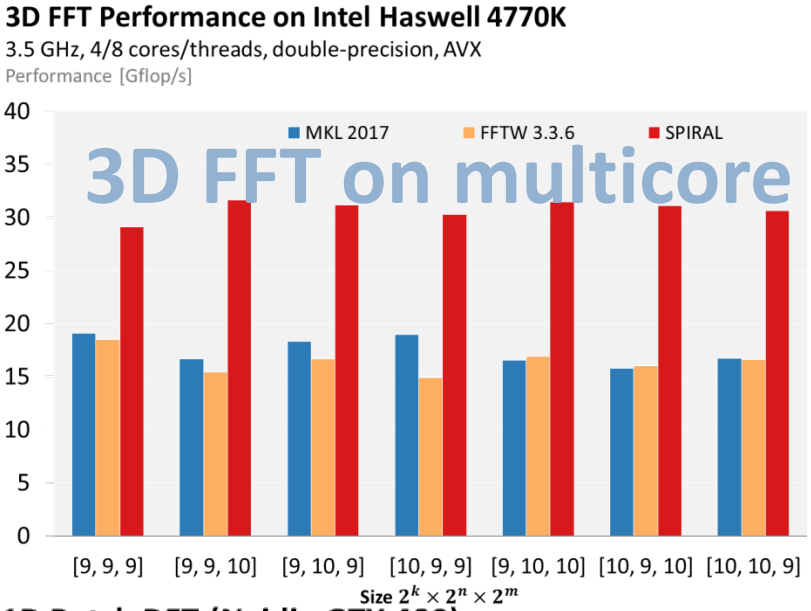
BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz

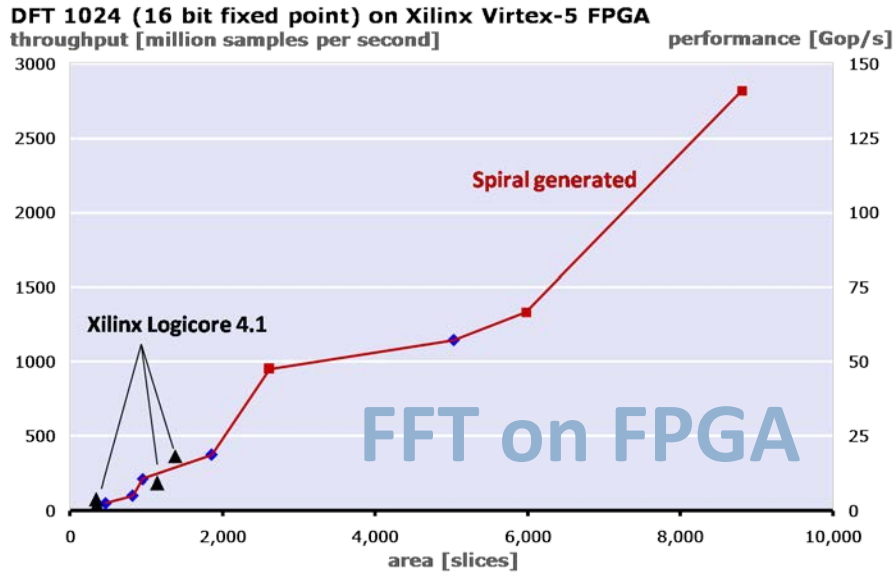
2006 Gordon Bell Prize (Peak Performance Award) with LLNL and IBM

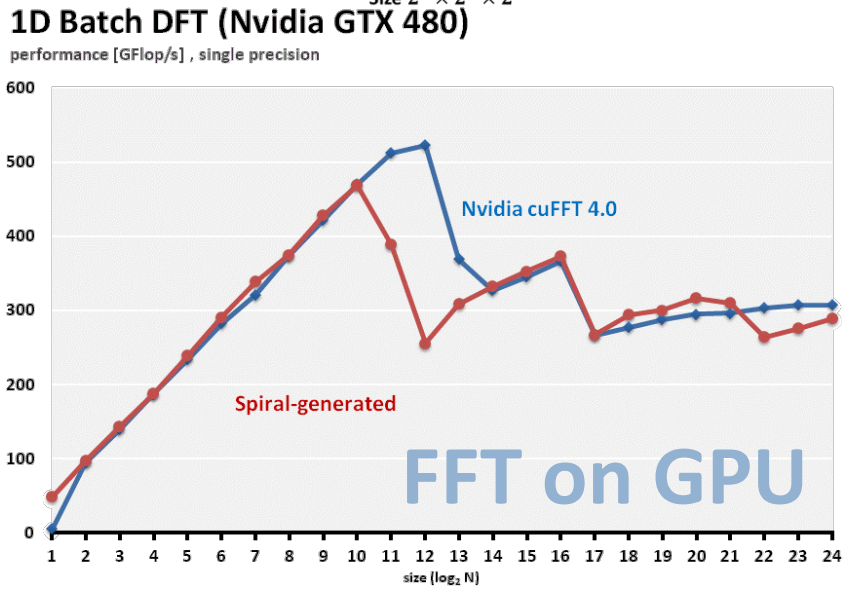
2010 HPC Challenge Class II Award (Most Productive System) with ANL and IBM

SPIRAL: Performance Across Platforms









Thrust 1: Repeat The Success of LAPACK for FFTs

Numerical Linear Algebra

LAPACK
ScaLAPACK
LU factorization
Eigensolves
SVD

BLAS, BLACS
BLAS-1
BLAS-2
BLAS-3

Spectral Algorithms

Convolution
Correlation
Upsampling
Poisson solver
…

?

FFTW
DFT, RDFT
1D, 2D, 3D,…
batch

No LAPACK equivalent for spectral methods

- **Medium size 1D FFT (1k—10k data points) is most common library call**
  applications break down 3D problems themselves and then call the 1D FFT library
- **Higher level FFT calls rarely used**
  FFTW *guru* interface is powerful but hard to used, leading to performance loss
- **Low arithmetic intensity and variation of FFT use make library approach hard**
  Algorithm specific decompositions and FFT calls intertwined with non-FFT code

Thrust 2: Updated FFTW Interface for Node FFTs

Front End: application facing

- **Backwards compatible to FFTW 2.X and 3.X**
  old code runs unmodified and gains substantially but not fully
- **Delayed execution/futures**
  asynchronous execution, offloading, triggering of execution
- **Data location and layout control**
  specify location of data (host/device, NUMA, NVRAM,…)
- **Performance metrics**
  user control of resource/performance/energy requirements and adaptation

Backend: developer and hardware facing

- **Extensible to novel and yet-unknown hardware and system architectures**
  future-proof platform extension API: transparently enable accelerators, etc.
- **Plug-in mechanism for special case acceleration packs**
  transparent access to application/architecture specialized FFT kernels

More Information:

www.spiral.net

www.spiralgen.com

We are about to make SPIRAL available as open source under a BSD style license