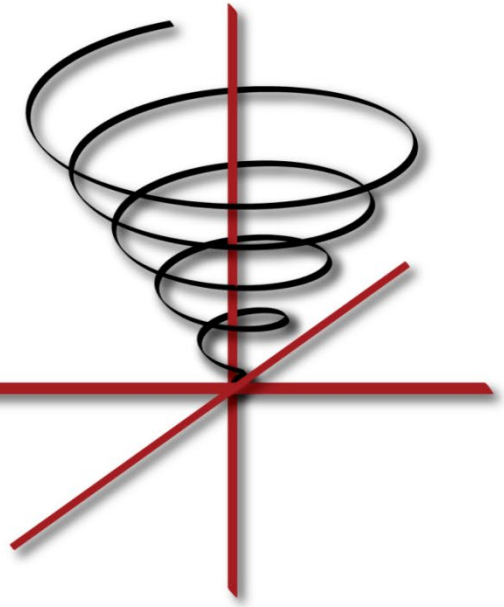
FFTX and SpectralPACK:
A First Look



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ECP EXASCALE COMPUTING PROJECT

Have You Ever Wondered About This?

Numerical Linear Algebra

LAPACK
ScaLAPACK
LU factorization
Eigensolves
SVD

BLAS, BLACS
BLAS-1
BLAS-2
BLAS-3

Spectral Algorithms

Convolution
Correlation
Upsampling
Poisson solver
…

?

FFTW
DFT, RDFT
1D, 2D, 3D,…
batch

No LAPACK equivalent for spectral methods

- **Medium size 1D FFT (1k—10k data points) is most common library call**
  applications break down 3D problems themselves and then call the 1D FFT library
- **Higher level FFT calls rarely used**
  FFTW *guru* interface is powerful but hard to used, leading to performance loss
- **Low arithmetic intensity and variation of FFT use make library approach hard**
  Algorithm specific decompositions and FFT calls intertwined with non-FFT code

It Is Worse Than It Seems

FFTW is de-facto standard interface for FFT

- **FFTW 3.X is the high performance reference implementation:** supports multicore/SMP and MPI, and Cell processor
- **Vendor libraries support the FFTW 3.X interface:** Intel MKL, IBM ESSL, AMD ACML (end-of-life), Nvidia cuFFT, Cray LibSci/CRAFFT

Issue 1: 1D FFTW call is standard kernel for many applications

- **Parallel libraries and applications reduce to 1D FFTW call** P3DFFT, QBox, PS/DNS, CPMD, HACC,…
- **Supported by modern languages and environments** Python, Matlab,…

Issue 2: FFTW is slowly becoming obsolete

- **FFTW 2.1.5 (still in use, 1997), FFTW 3 (2004) minor updates since then**
- **Development currently dormant, except for small bug fixes**
- **No native support for accelerators (GPUs, Xeon PHI, FPGAs) and SIMT**
- **Parallel/MPI version does not scale beyond 32 nodes**

Risk: loss of high performance FFT standard library

FFTX: The FFTW Revamp for ExaScale

Modernized FFTW-style interface

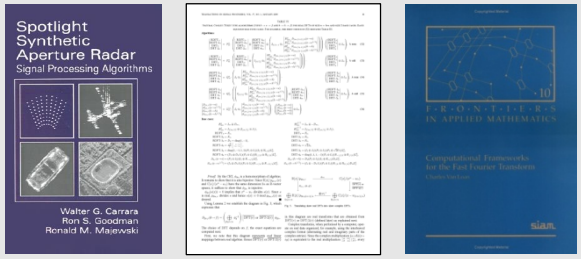
- **Backwards compatible to FFTW 2.X and 3.X**
  old code runs unmodified and gains substantially but not fully
- **Small number of new features**
  futures/delayed execution, offloading, data placement, callback kernels

Code generation backend using SPIRAL

- **Library/application kernels are interpreted as specifications in DSL**
  extract semantics from source code and known library semantics
- **Compilation and advanced performance optimization**
  cross-call and cross library optimization, accelerator off-loading,…
- **Fine control over resource expenditure of optimization**
  compile-time, initialization-time, invocation time, optimization resources
- **Reference library implementation and bindings to vendor libraries**
  library-only reference implementation for ease of development

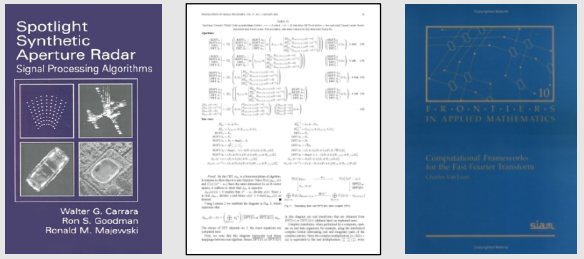
What is Spiral? Autotuning/Code Generation

Traditionally



High performance library optimized for given platform

Spiral Approach

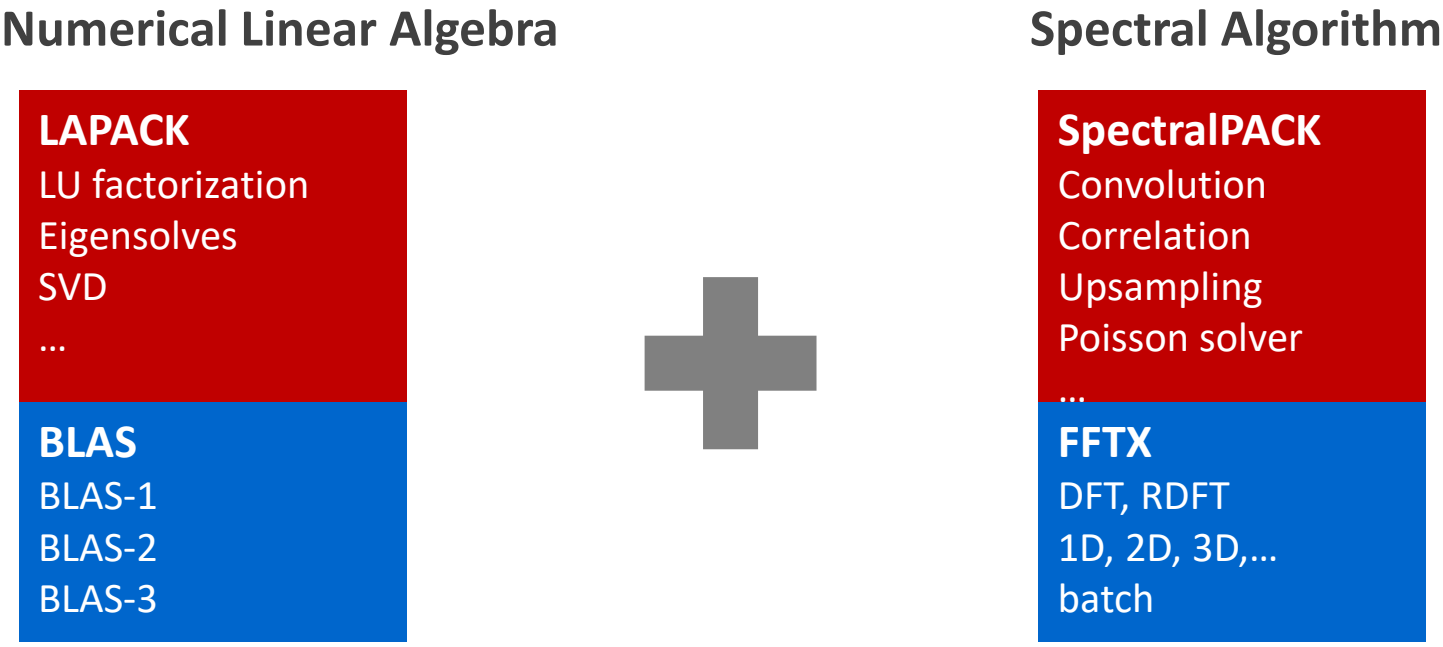


Spiral

High performance library optimized for given platform

Comparable performance

FFTX and SpectralPACK: Long Term Vision



Define the LAPACK equivalent for spectral algorithms

- **Define FFTX as the BLAS equivalent**
  provide user FFT functionality as well as algorithm building blocks
- **Define class of numerical algorithms to be supported by SpectralPACK**
  PDE solver classes (Green's function, sparse in normal/k space,…), signal processing,…
- **Define SpectralPACK functions**
  circular convolutions, NUFFT, Poisson solvers, free space convolution,…

FFTX and SpectralPACK solve the "spectral dwarf" long term

Poisson's Equation in Free Space: The Math

Partial differential equation (PDE)

$$\Delta(\Phi) = \rho$$

$$\rho : \mathbb{R}^3 \to \mathbb{R}$$

$$D = \text{supp}(\rho) \subset \mathbb{R}^3$$

Poisson's equation. Δ is the Laplace operator

Solution characterization

$$\Phi : \mathbb{R}^3 \to \mathbb{R}$$

$$\Phi(\vec{x}) = \frac{Q}{4\pi||\vec{x}||} + o\left(\frac{1}{||\vec{x}||}\right) \text{ as } ||\vec{x}|| \to \infty$$

$$Q = \int_D \rho d\vec{x}$$

Approach: Green's function

$$\Phi(\vec{x}) = \int_D G(\vec{x} - \vec{y})\rho(\vec{y})d\vec{y} \equiv (G * \rho)(\vec{x}), \quad G(\vec{x}) = \frac{1}{4\pi||\vec{x}||_2}$$

Solution: ϕ(.) = convolution of RHS *ρ*(.) with Green's function *G*(.). Efficient through FFTs (frequency domain)

Method of Local Corrections (MLC)

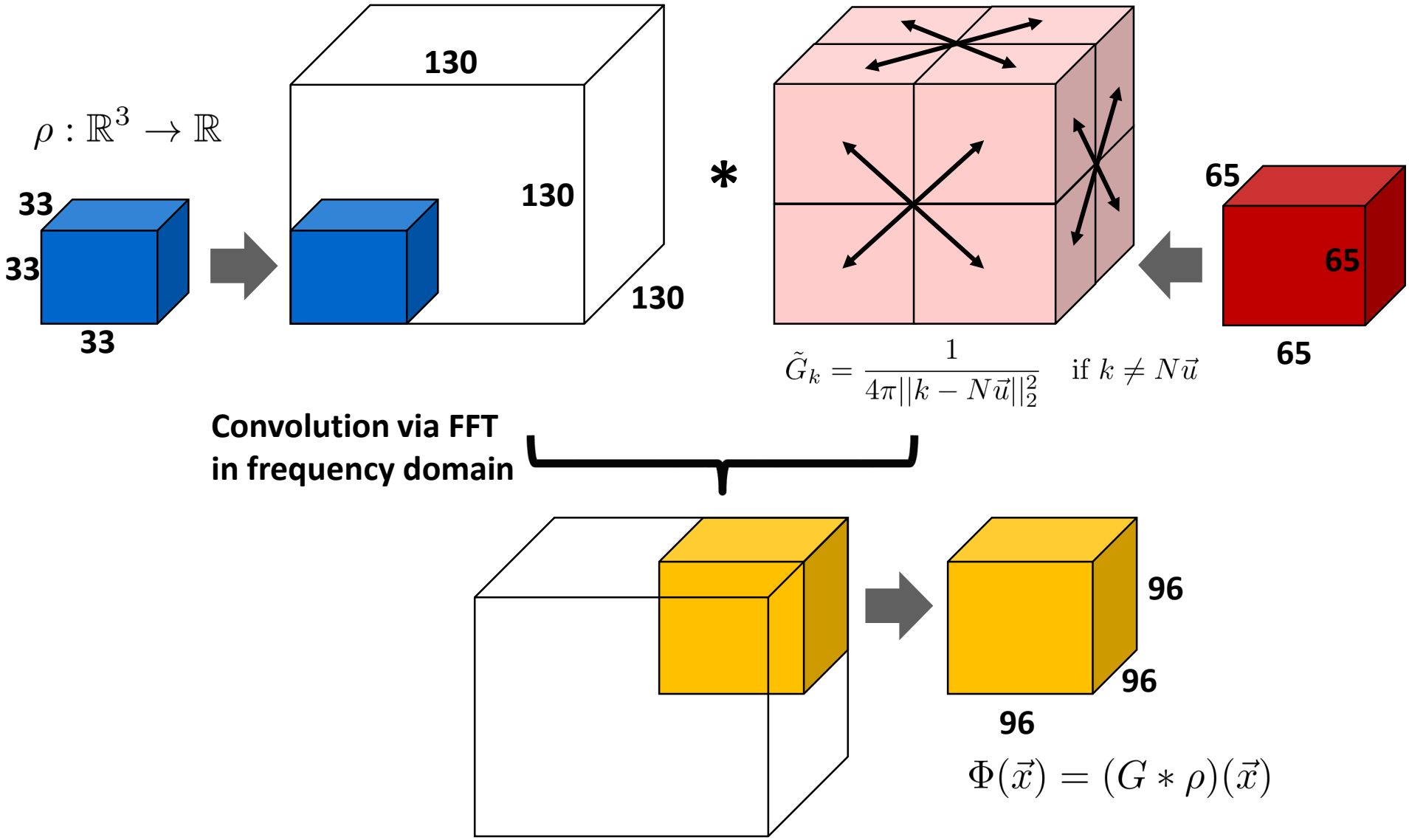
$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

Green's function kernel in frequency domain

P. McCorquodale, P. Colella, G. T. Balls, and S. B. Baden, "A local corrections algorithm for solving Poisson's equation in three dimensions," vol. 2, 10 2006.

C. R. Anderson, "A method of local corrections for computing the velocity field due to a distribution of vortex blobs," Journal of Computational Physics, vol. 62, no. 1, pp. 111–123, 1986.

Algorithm: Hockney Free Space Convolution



Hockney: Convolution + problem specific zero padding and output subset

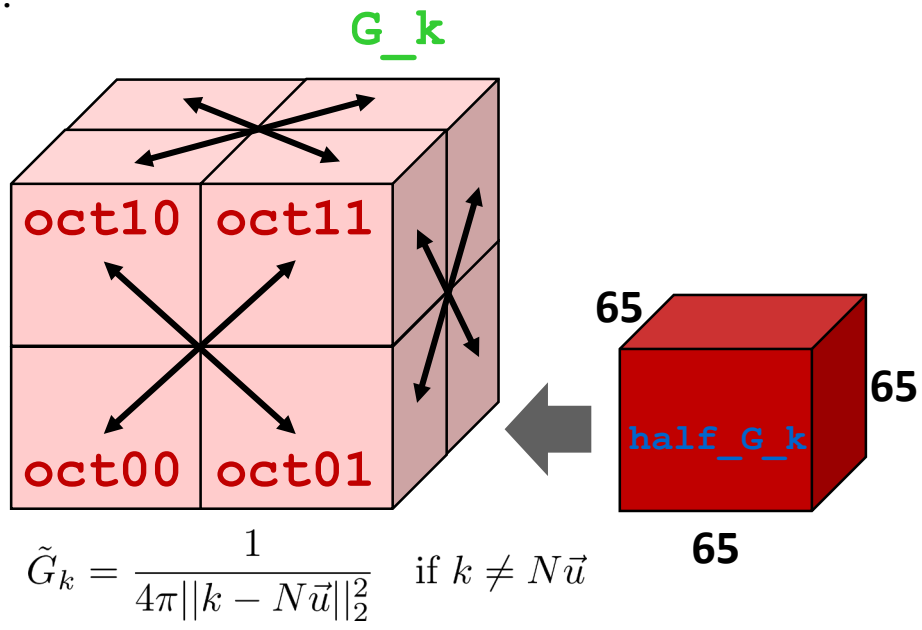
FFTX Example: Hockney Free Space Convolution

```
fftx_plan pruned_real_convolution_plan(fftx_real *in, fftx_real *out, fftx_complex *symbol,
        int n, int n_in, int n_out, int n_freq) {
    int rank = 3,
    batch_rank = 0,
    ...
    fftx_plan plans[5];
    fftx_plan p;

    tmp1 = fftx_create_zero_temp_real(rank, &padded_dims);

    plans[0] = fftx_plan_guru_copy_real(rank, &in_dimx, in, tmp1, MY_FFTX_MODE_SUB);

    tmp2 = fftx_create_temp_complex(rank, &freq_dims);
    plans[1] = fftx_plan_guru_dft_r2c(rank, &padded_dims, batch_rank,
        &batch_dims, tmp1, tmp2, MY_FFTX_MODE_SUB);

    tmp3 = fftx_create_temp_complex(rank, &freq_dims);
    plans[2] = fftx_plan_guru_pointwise_c2c(rank, &freq_dimx, batch_rank, &batch_dimx,
        tmp2, tmp3, symbol, (fftx_callback)complex_scaling,
        MY_FFTX_MODE_SUB | FFTX_PW_POINTWISE);

    tmp4 = fftx_create_temp_real(rank, &padded_dims);
    plans[3] = fftx_plan_guru_dft_c2r(rank, &padded_dims, batch_rank,
        &batch_dims, tmp3, tmp4, MY_FFTX_MODE_SUB);

    plans[4] = fftx_plan_guru_copy_real(rank, &out_dimx, tmp4, out, MY_FFTX_MODE_SUB);

    p = fftx_plan_compose(numsubplans, plans, MY_FFTX_MODE_TOP);

    return p;
}
```

Looks like FFTW calls, but is a specification for SPIRAL

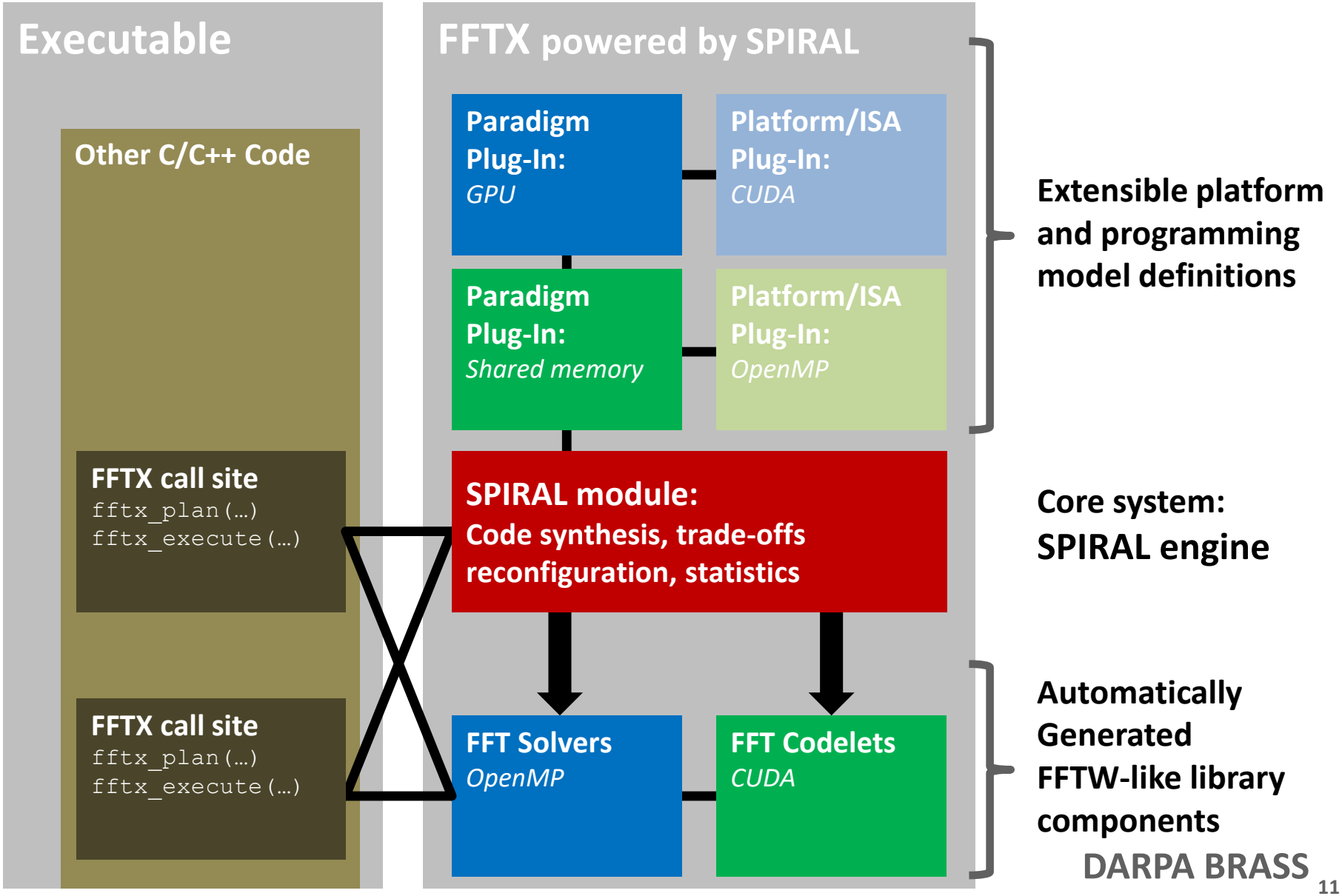
FFTX Example: Describing The Hockney Symmetry

```
// FFTX data access descriptors.
// Access is to four octants of a symmetric cube.
// Cube size is N^3 and M = N/2.

fftx_iodimx oct00[] = {
  { M+1, 0, 0, 0, 1, 1, 1 },
  { M+1, 0, 0, 0, M+1, 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} },
  oct01[] = {
  { M-1, M-1, M+1, 0, -1, 1, 1 },
  { M+1, 0, 0, 0, M+1, 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} },
  oct10[] = {
  { M+1, 0, 0, 0, 1, 1, 1 },
  { M-1, M-1, M+1, 0, -(M+1), 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} },
 oct11[] = {
  { M-1, M-1, M+1, 0, -1, 1, 1 },
  { M-1, M-1, M+1, 0, -(M+1), 2*M, 1 },
  { M+1, 0, 0, 0, (M+1)*(M+1), 4*M*M, 1} };
...

fftx_temp_complex half_G_k = fftx_create_zero_temp_complex(rk, f_d);
plans[2] = fftx_plan_guru_copy_complex(rk, oct00, G_k, half_G_k, FFTX_MODE_SUB);
plans[3] = fftx_plan_guru_copy_complex(rk, oct01, G_k, half_G_k, MY_FFTX_MODE_SUB);
plans[4] = fftx_plan_guru_copy_complex(rk, oct10, G_k, half_G_k, MY_FFTX_MODE_SUB);
plans[5] = fftx_plan_guru_copy_complex(rk, oct11, G_k, half_G_k, MY_FFTX_MODE_SUB);
...
```



$$\tilde{G}_k = \frac{1}{4\pi||k - N\vec{u}||_2^2} \quad \text{if } k \neq N\vec{u}$$

FFTX Backend: SPIRAL



Generated Code For Hockney Convolution

```
void ioprunedconv_130_0_62_72_130(double  *Y, double  *X, double * S) {
    static double D84[260] = {65.5, 0.0, (-0.50000000000001132), (-20.686114762237267),
      (-0.5000000000000081), (-10.337014680426078), (-0.50000000000000455),
    ...
for(int i18899 = 0; i18899 <= 1; i18899++) {
    for(int i18912 = 0; i18912 <= 4; i18912++) {
        a9807 = ((2*i18899) + (4*i18912));
        a9808 = (a9807 + 1);
        a9809 = (a9807 + 52);
        a9810 = (a9807 + 53);
        a9811 = (a9807 + 104);
        a9812 = (a9807 + 105);
        s3295 = (*((X + a9807)) + *((X + a9809))
            + *((X + a9811)));
        s3296 = (*((X + a9808)) + *((X + a9810))
            + *((X + a9812)));
        s3297 = (((0.30901699437494474**((X + a9809)))
            - (0.80901699437494745**((X + a9811))))
            + *((X + a9807)));
        ...
        *((104 + Y + a12569)) = ((s3983 - s3987)
            + (0.80901699437494745*t6537)
            + (0.58778525229247314*t6538));
        *((105 + Y + a12569)) = (((s3984 - s3988)
            + (0.80901699437494745*t6538))
            - (0.58778525229247314*t6537));
    }
}
```

FFTX/SPIRAL with OpenACC backend
Compared to cuFFT expert interface

15% faster on TITAN V

Same speed on Tesla V100

1,000s of lines of code, cross call optimization, etc., transparently used

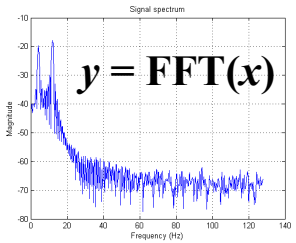
SPIRAL: Go from Mathematics to Software

Given:

- **Mathematical problem specification**
  does not change
- **Computer platform**
  changes often

Wanted:

- **Very good implementation of specification on platform**
- **Proof of correctness**



on

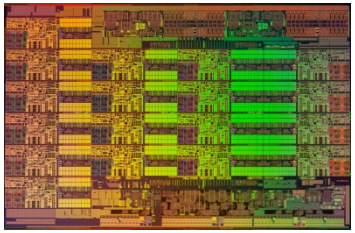
automatic

```
void fft64(double  *Y, double  *X) {
    ...
    s5674 = _mm256_permute2f128_pd(s5672, s5673, (0) | ((2) << 4));
    s5675 = _mm256_permute2f128_pd(s5672, s5673, (1) | ((3) << 4));
    s5676 = _mm256_unpacklo_pd(s5674, s5675);
    s5677 = _mm256_unpackhi_pd(s5674, s5675);
    s5678 = *((a3738 + 16));
    s5679 = *((a3738 + 17));
    s5680 = _mm256_permute2f128_pd(s5678, s5679, (0) | ((2) << 4));
    s5681 = _mm256_permute2f128_pd(s5678, s5679, (1) | ((3) << 4));
    s5682 = _mm256_unpacklo_pd(s5680, s5681);
    s5683 = _mm256_unpackhi_pd(s5680, s5681);
    t5735 = _mm256_add_pd(s5676, s5682);
    t5736 = _mm256_add_pd(s5677, s5683);
    t5737 = _mm256_add_pd(s5670, t5735);
    t5738 = _mm256_add_pd(s5671, t5736);
    t5739 = _mm256_sub_pd(s5670, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5735));
    t5740 = _mm256_sub_pd(s5671, _mm256_mul_pd(_mm_vbroadcast_sd(&(C22)), t5736));
    t5741 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5677, s5683));
    t5742 = _mm256_mul_pd(_mm_vbroadcast_sd(&(C23)), _mm256_sub_pd(s5676, s5682));
    ...
}
```

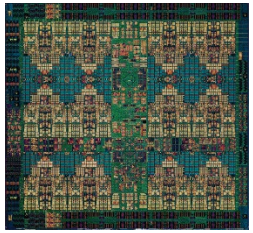
QED.

SPIRAL's Target Computing Landscape

1 Gflop/s = one billion floating-point operations (additions or multiplications) per second

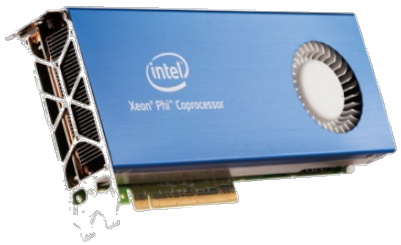


Intel Xeon 8180M
2.25 Tflop/s, 205 W
28 cores, 2.5—3.8 GHz
2-way—16-way AVX-512



IBM POWER9
768 Gflop/s, 300 W
24 cores, 4 GHz
4-way VSX-3

Nvidia Tesla V100
7.8 Tflop/s, 300 W
5120 cores, 1.2 GHz
32-way SIMT



Intel Xeon Phi 7290F
1.7 Tflop/s, 260 W
72 cores, 1.5 GHz
8-way/16-way LRBni

Snapdragon 835
15 Gflop/s, 2 W
8 cores, 2.3 GHz
A540 GPU, 682 DSP, NEON

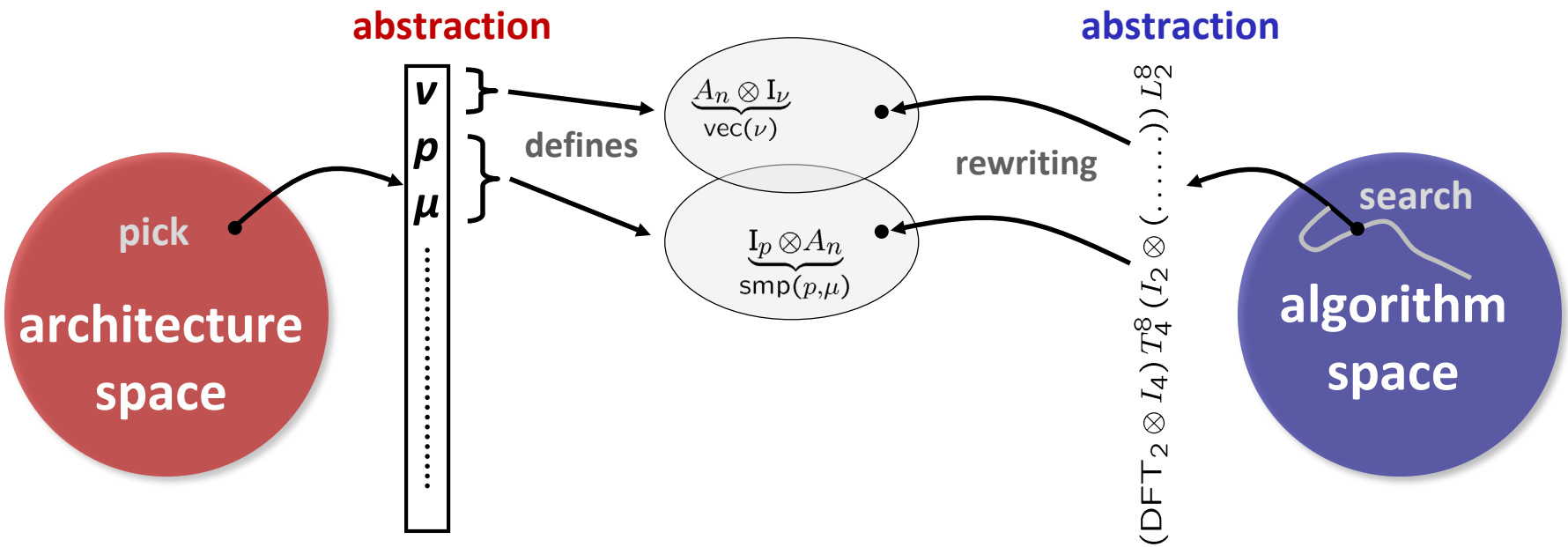
Intel Atom C3858
32 Gflop/s, 25 W
16 cores, 2.0 GHz
2-way/4-way SSSE3

Dell PowerEdge R940
3.2 Tflop/s, 6 TB, 850 W
4x 24 cores, 2.1 GHz
4-way/8-way AVX

Summit
187.7 Pflop/s, 13 MW
9,216 x 22 cores POWER9
+ 27,648 V100 GPUs

Platform-Aware Formal Program Synthesis

Model: common abstraction
= spaces of matching formulas



Architectural parameter:
Vector length,
#processors, …

optimization

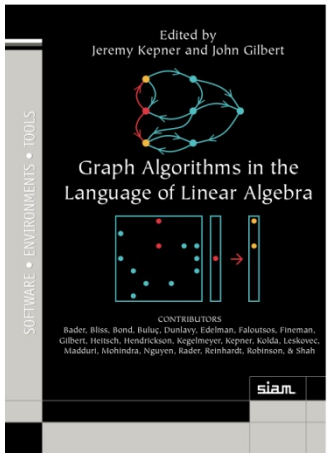
Kernel:
problem size,
algorithm choice

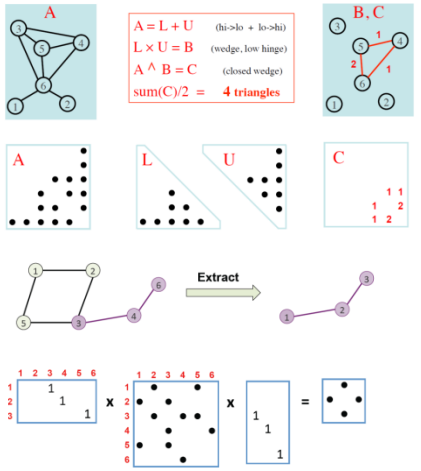
Algorithms: Rules in Domain Specific Language

Linear Transforms

$$\begin{aligned}
\mathbf{DFT}_n &\rightarrow (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\rightarrow P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m)Q_n, \quad n = km,\ \gcd(k,m) = 1 \\
\mathbf{DFT}_p &\rightarrow R_p^T(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})D_p(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})R_p, \quad p \text{ prime} \\
\mathbf{DCT\text{-}3}_n &\rightarrow (\mathrm{I}_m \oplus \mathrm{J}_m)\, \mathrm{L}^n_m(\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \\
&\quad \cdot(\mathrm{F}_2 \otimes \mathrm{I}_m) \begin{bmatrix} \mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1} \\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m) \end{bmatrix}, \quad n = 2m \\
\mathbf{DCT\text{-}4}_n &\rightarrow S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\rightarrow (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m)\left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right)\right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m} \\
\mathbf{WHT}_{2^k} &\rightarrow \prod_{i=1}^{t} (\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t \\
\mathbf{DFT}_2 &\rightarrow \mathrm{F}_2 \\
\mathbf{DCT\text{-}2}_2 &\rightarrow \mathrm{diag}(1, 1/\sqrt{2})\, \mathrm{F}_2 \\
\mathbf{DCT\text{-}4}_2 &\rightarrow \mathrm{J}_2 \mathrm{R}_{13\pi/8}
\end{aligned}$$

Graph Algorithms



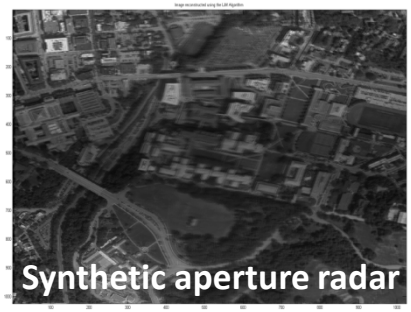


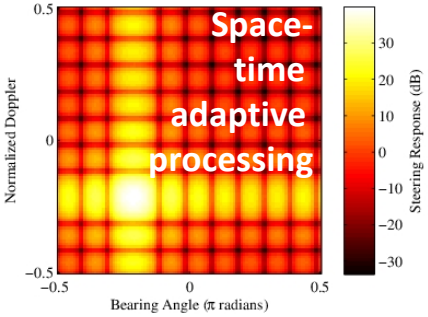
In collaboration with CMU-SEI

Numerical Linear Algebra

$$\begin{aligned}
&\mathrm{MMM}_{1,1,1} \rightarrow (\cdot)_1 \\
&\mathrm{MMM}_{m,n,k} \rightarrow (\otimes)_{m/m_b \times 1} \otimes \mathrm{MMM}_{m_b,n,k} \\
&\mathrm{MMM}_{m,n,k} \rightarrow \mathrm{MMM}_{m,nb,k} \otimes (\otimes)_{1 \times n/nb} \\
&\mathrm{MMM}_{m,n,k} \rightarrow ((\Sigma_{k/k_b} \circ (\cdot)_{k/k_b}) \otimes \mathrm{MMM}_{m,n,k_b}) \circ \\
&\qquad ((L^{mk/k_b}_{k/k_b} \otimes I_{k_b}) \times I_{kn}) \\
&\mathrm{MMM}_{m,n,k} \rightarrow (L^{mn/n_b}_m \otimes I_{n_b}) \circ \\
&\qquad ((\otimes)_{1 \times n/n_b} \otimes \mathrm{MMM}_{m,n_b,k}) \circ \\
&\qquad (I_{km} \times (L^{kn/n_b}_{n/n_b} \otimes I_{n_b}))
\end{aligned}$$

Spectral Domain Applications





Formal Approach for all Types of Parallelism

- **Multithreading** (Multicore)
- **Vector SIMD** (SSE, VMX/Altivec,…)
- **Message Passing** (Clusters, MPP)
- **Streaming/multibuffering** (Cell)
- **Graphics Processors** (GPUs)
- **Gate-level parallelism** (FPGA)
- **HW/SW partitioning** (CPU + FPGA)

$$\mathrm{I}_p \otimes_{\|} A_{\mu n}, \quad \mathbf{L}^{mn}_m \bar{\otimes} \mathrm{I}_\mu$$

$$A \hat{\otimes} \mathrm{I}_\nu \quad \underbrace{\mathbf{L}^{2\nu}_2}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{2\nu}_\nu}_{\text{isa}}, \quad \underbrace{\mathbf{L}^{\nu^2}_\nu}_{\text{isa}}$$

$$\mathrm{I}_p \otimes_{\|} A_n, \quad \underbrace{\mathbf{L}^{p^2}_p \bar{\otimes} \mathrm{I}_{n/p^2}}_{\text{all-to-all}}$$

$$\mathrm{I}_n \otimes_2 A_{\mu n}, \quad \mathbf{L}^{mn}_m \bar{\otimes} \mathrm{I}_\mu$$

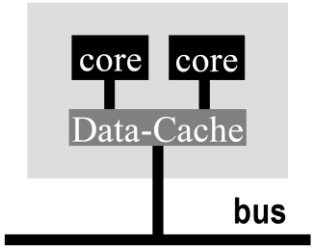
$$\prod_{i=0}^{n-1} A_i, \quad A_n \hat{\otimes} \mathrm{I}_w, \quad P_n \otimes Q_w$$

$$\prod_{i=0}^{n-1^{\mathsf{ir}}} A, \quad \mathrm{I}_s \tilde{\otimes} A, \quad \underbrace{\mathbf{L}^m_n}_{\text{bram}}$$

$$\underbrace{A_1}_{\text{fpga}}, \quad \underbrace{A_2}_{\text{fpga}}, \quad \underbrace{A_3}_{\text{fpga}}, \quad \underbrace{A_4}_{\text{fpga}}$$

Modeling Hardware: Base Cases

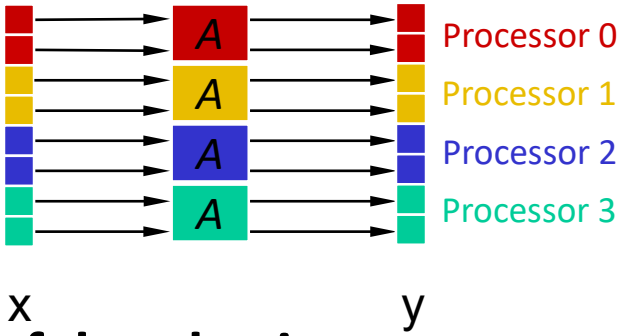
- **Hardware abstraction: shared cache with cache lines**



$$\underbrace{A_{k\times m\to n}}_{\mathrm{smp}(p,\mu)}$$

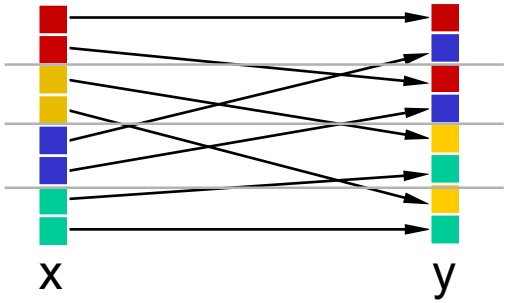
- **Tensor product: embarrassingly parallel operator**

$$y = \left(\mathrm{I}_p \otimes A\right)(x)$$

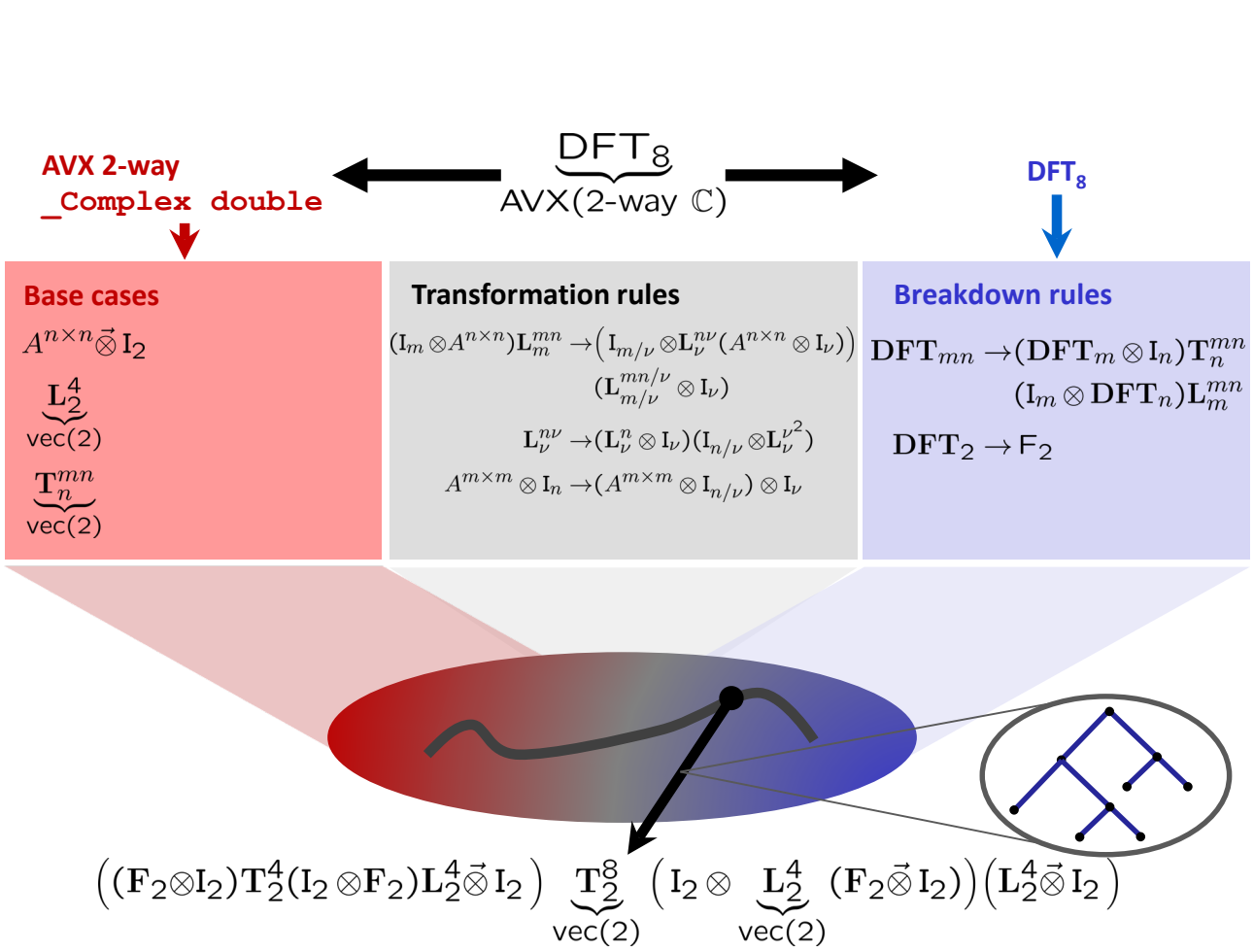


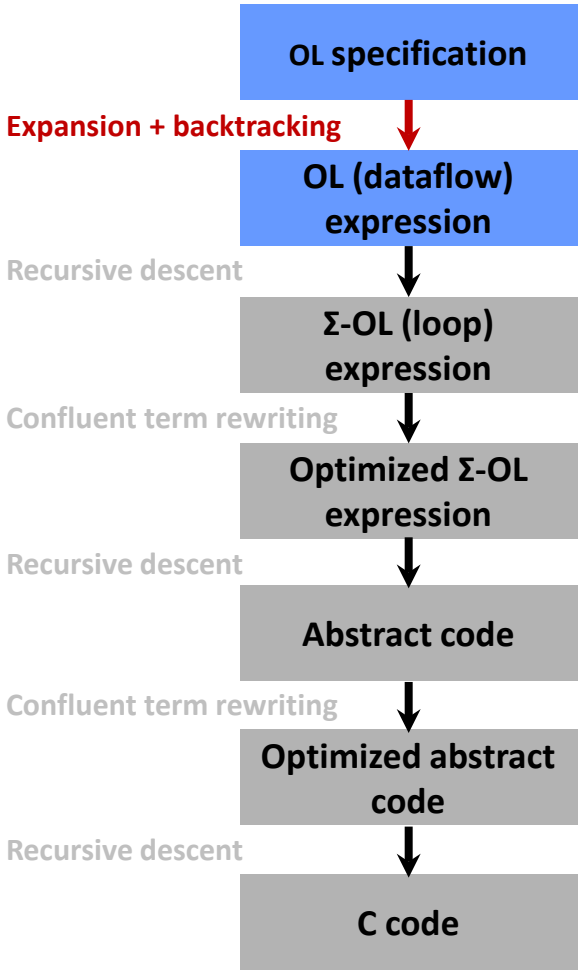
- **Permutation: problematic; may produce false sharing**

$$y = \mathrm{L}^{8}_{4}(x)$$



Autotuning in Constraint Solution Space





Translating an OL Expression Into Code

Constraint Solver Input: $\underbrace{DFT_8}_{AVX(2\text{-way } \mathbb{C})}$

Output =

Ruletree, expanded into

See Figure 5

OL Expression:

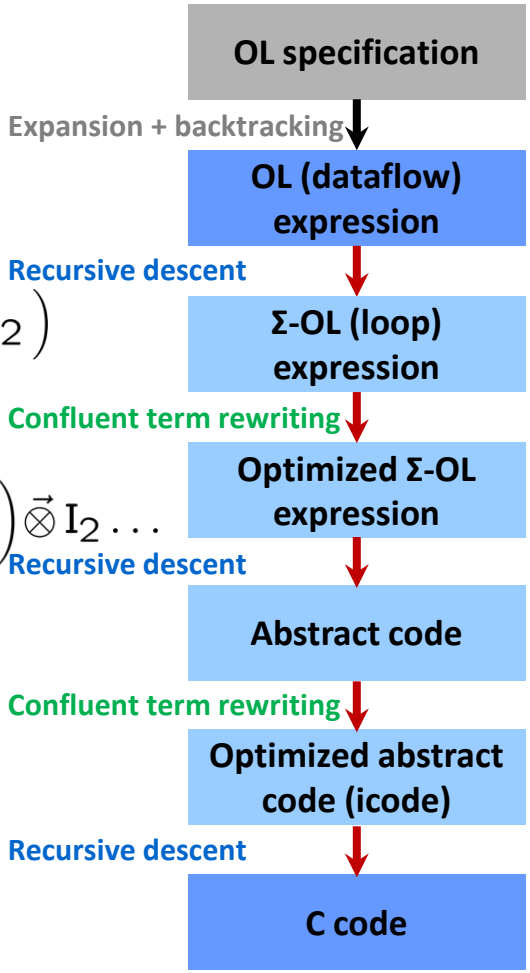
$$\left((F_2 \otimes I_2) T^4_2 (I_2 \otimes F_2) L^4_2 \vec{\otimes} I_2\right) \underbrace{T^8_2}_{vec(2)} \left(I_2 \otimes \underbrace{L^4_2}_{vec(2)} (F_2 \vec{\otimes} I_2)\right)\left(L^4_2 \vec{\otimes} I_2\right)$$

∑-OL:

$$\left(\sum_{j=0}^{1} \left(S_{\imath_2 \otimes (j)_2} F_2 \mathrm{Map}^2_{x \mapsto \omega_4^{2i+j} x} G_{\imath_2 \otimes (j)_2}\right) \sum_{j=0}^{1} \left(S_{(j)_2 \otimes \imath_2} F_2 G_{\imath_2 \otimes (j)_2}\right)\right) \vec{\otimes} I_2 \ldots$$

C Code:

```
void dft8(_Complex double *Y, _Complex double *X) {
    __m256d s38, s39, s40, s41,...
    __m256d  *a17, *a18;
    a17 = ((__m256d  *) X);
    s38 = *(a17);
    s39 = *((a17 + 2));
    t38 = _mm256_add_pd(s38, s39);
    t39 = _mm256_sub_pd(s38, s39);
    ...
    s52 = _mm256_sub_pd(s45, s50);
    *((a18 + 3)) = s52;
}
```



Symbolic Verification for Linear Operators

- **Linear operator = matrix-vector product**
  Algorithm = matrix factorization

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} = \begin{bmatrix} 1 & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & 1 \\ 1 & \cdot & -1 & \cdot \\ \cdot & 1 & \cdot & -1 \end{bmatrix} \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & j \end{bmatrix} \begin{bmatrix} 1 & 1 & \cdot & \cdot \\ 1 & -1 & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

= ?

$$\mathrm{DFT}_4 = (\mathrm{DFT}_2 \otimes \mathrm{I}_2)\, \mathrm{T}^4_2 (\mathrm{I}_2 \otimes \mathrm{DFT}_2)\, \mathrm{L}^4_2$$

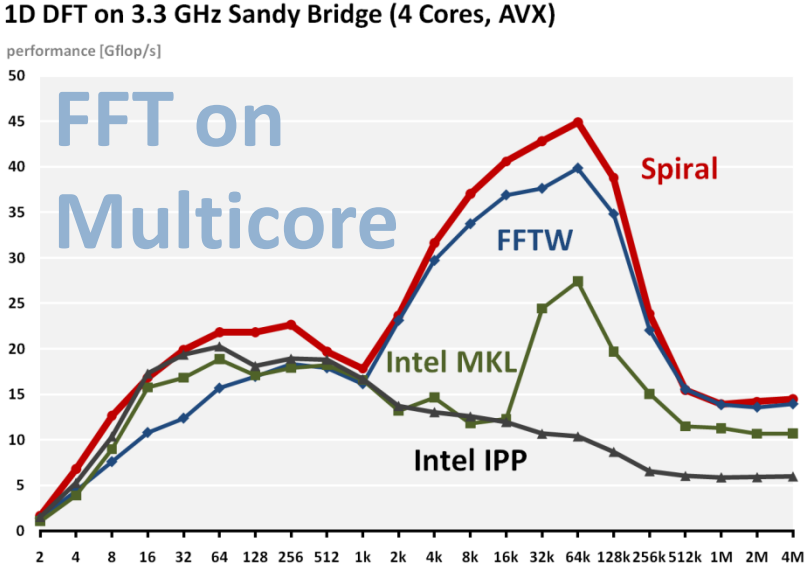
- **Linear operator = matrix-vector product**
  Program = matrix-vector product

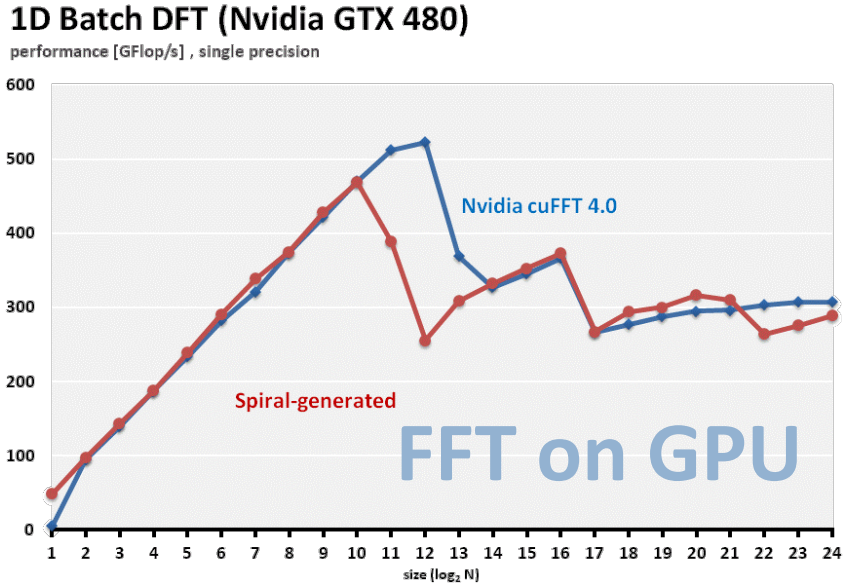
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

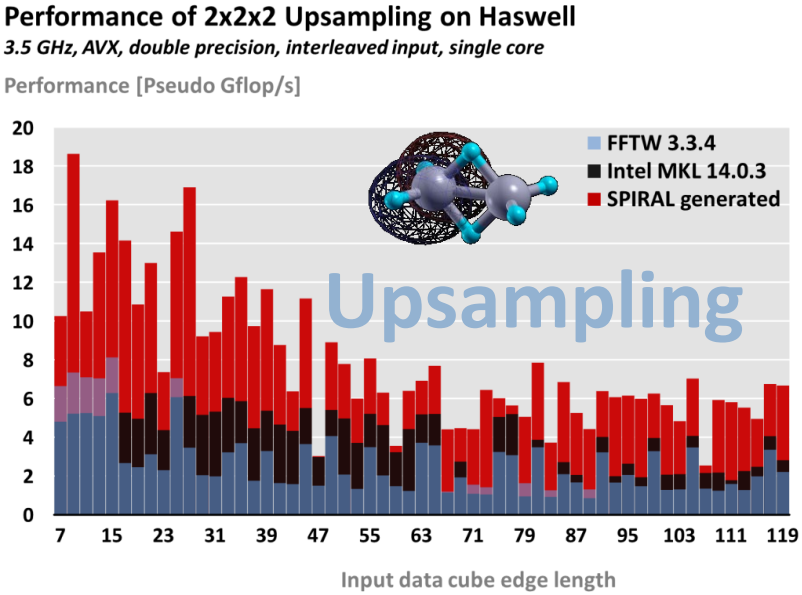
= ? `DFT4([0,1,0,0])`

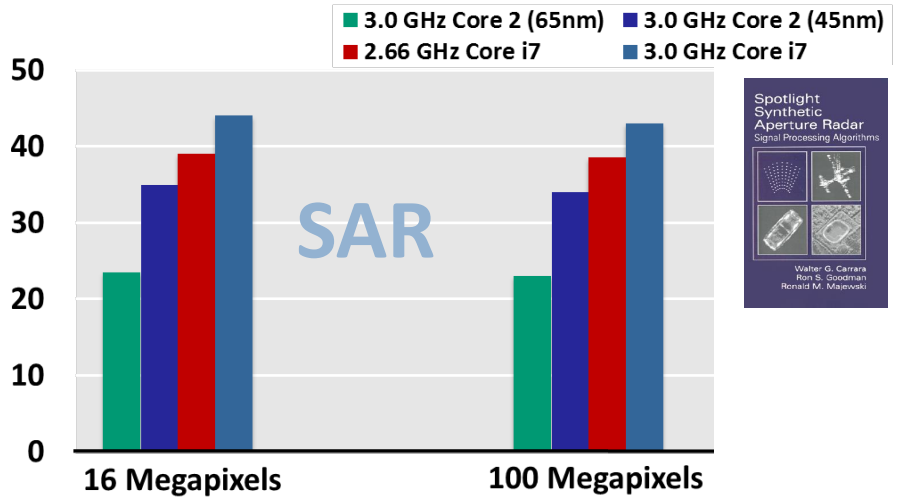
Symbolic evaluation and symbolic execution establishes correctness

Results: FFTs and Spectral Algorithms





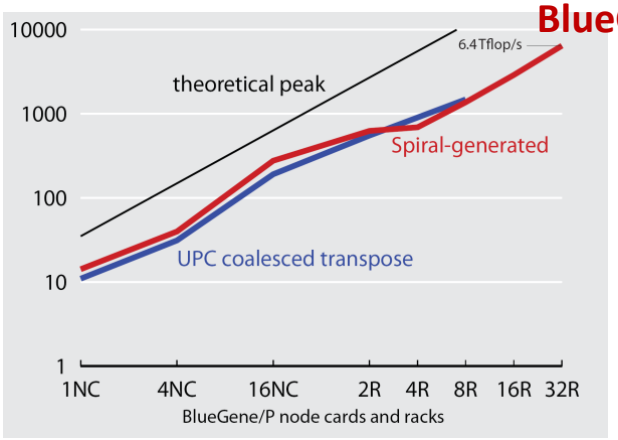




SPIRAL: Success in HPC/Supercomputing

- **NCSA Blue Waters**
  PAID Program, FFTs for Blue Waters
- **RIKEN K computer**
  FFTs for the HPC-ACE ISA
- **LANL RoadRunner**
  FFTs for the Cell processor
- **PSC/XSEDE Bridges**
  Large size FFTs
- **LLNL BlueGene/L and P**
  FFTW for BlueGene/L's Double FPU
- **ANL BlueGene/Q Mira**
  Early Science Program, FFTW for BGQ QPX

Global FFT (1D FFT, HPC Challenge)



6.4 Tflop/s on BlueGene/P

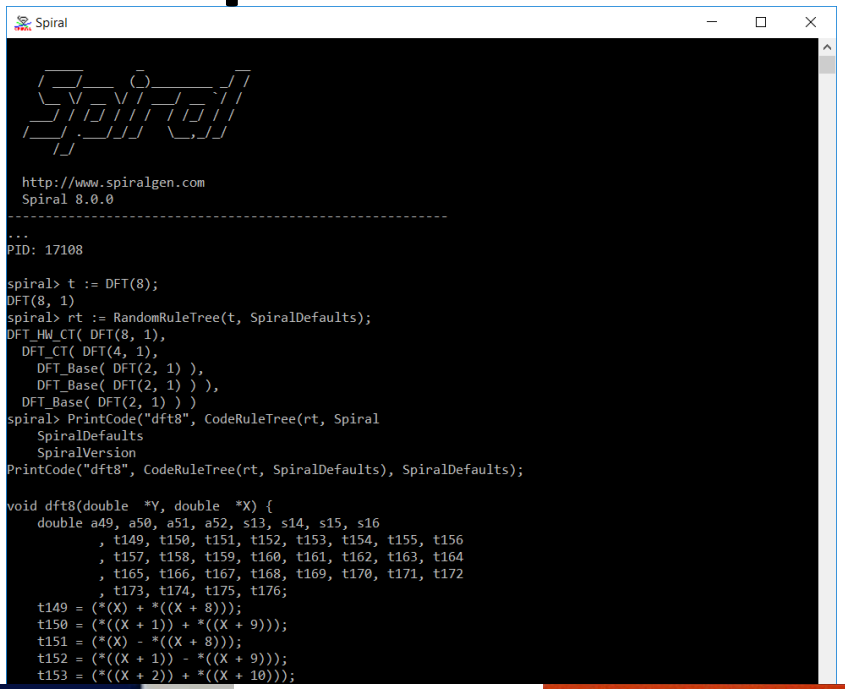
BlueGene/P at Argonne National Laboratory
128k cores (quad-core CPUs) at 850 MHz

2006 Gordon Bell Prize (Peak Performance Award) with LLNL and IBM

2010 HPC Challenge Class II Award (Most Productive System) with ANL and IBM

SPIRAL 8.0: Available Under Open Source

- **Open Source SPIRAL available**
  - non-viral license (BSD)
  - Initial version, effort ongoing to open source whole system
  - Commercial support via SpiralGen, Inc.
- **Developed over 20 years**
  - Funding: DARPA (OPAL, DESA, HACMS, PERFECT, BRASS), NSF, ONR, DoD HPC, JPL, DOE, CMU SEI, Intel, Nvidia, Mercury
- **Open sourced under DARPA PERFECT**



www.spiral.net

F. Franchetti, T. M. Low, D. T. Popovici, R. M. Veras, D. G. Spampinato, J. R. Johnson, M. Püschel, J. C. Hoe, J. M. F. Moura:
SPIRAL: Extreme Performance Portability, Proceedings of the IEEE, Vol. 106, No. 11, 2018.
Special Issue on *From High Level Specification to High Performance Code*